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Example proposition

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Abstract

This article is not so much about quantum logic. It uses quantum logic because traditional quantum logic defines the static framework in which quantum dynamics takes place. This framework is represented by the set of closed subspaces of a Hilbert space. The article reveals that there is a need to extend traditional quantum logic such that it includes axioms, which specify the dynamic underpinning of quantum physics. In the course of this project several fundamental aspects of physics get uncovered.

Quantum logic is usually defined as an orthomodular lattice. It only defines the *static* skeleton of the frame in which quantum physics operates. It also does not state anything about physical fields. Inertia reveals the importance of fields. The Helmholtz/Hodge decomposition theorem defines the stationary structure of these fields. In that way this theorem plays a similar role as traditional quantum logic. These law sets do not specify or even touch the source of *dynamics*.

Both the propositions about a quantum physical system and physical fields are closely related. However, this relation gets relevant when dynamics comes into play. Dynamics causes a continuing redefinition of the propositions. This disturbs the static status quo. When one proposition is changed it interchanges its constituting atomic propositions with other propositions. The change can even involve the exchange of atomic propositions against atomic propositions that are of another type. It is also possible that the configuration of a complex system that consists of simpler components is altered.

The physical fields are the representatives of the influences that go together by the sticky resistance of the set of propositions against the changes that occur due to the redefinitions of the propositions that describe physical items. This sticky resistance also occurs in propositions that are sub-ordered to other propositions. Inertia is a feature that shows this resistance explicitly.

The propositions about quantum physical items can be represented by closed subspaces of a Hilbert space. The presence of dynamics means that the relations between these subspaces are not stationary. It is also possible to give the physical fields a representation in Hilbert space. However, it must be clear that quantum physical items and physical fields are not the same stuff. Physical fields cannot be represented by closed Hilbert subspaces. They cover the whole universe. However, their strength may be concentrated at separate places and it may diminish with distance.

It is an elucidating experience to try to implement a complicated quantum logical proposition in the representation of quantum logic in Hilbert space. We want to see how dynamics emerges in this static skeleton. For that reason, we choose as an example a predicate with quantifiers rather than a clean proposition. In the course of this project it will become clear

that there is a way to extend the rather static traditional quantum logic into a dynamic version. The phenomenon of inertia guides the way.

The selected example proposition (*) is

“All items in universe influence each other’s position”.

The final conclusion of this reasoning is: A well-ordered replacement of atomic propositions in an enveloping proposition appears to occur without strong consequences, but any deviation of a well ordered replacement causes an influence of the complete set of all propositions. This explains the interaction between fields and physical items. A local deviation of the uniformity of the distribution of physical items can still cause a slight influence of neighboring items. At small distances the influences can be large. The influence of fields can be implemented in the Hilbert space. Via an action = reaction game the interaction between fields and Hilbert subspaces form the source of dynamics.

What further happens during the implementation of our example proposition (*) is completely governed by mathematics. Thus, no further extension of quantum logic is needed to transform it *for our example* into a useful version of dynamic quantum logic.

As number fields we use the 2^n -ons of Warren Smith rather than the hyper complex numbers based on the Cayley-Dickson construction. Up to the octonions the corresponding number fields are similar. For higher n the 2^n -ons behave in a nicer way. We use the quaternions ($n=2$) as the number field that is used to define the inner product of the Hilbert space. However, we tolerate operators to have eigenvalues that are higher dimensional 2^n -ons. In this respect, we apply their storage capacity rather than their number characteristics. A 2^n -on contains 2^n real numbers. Scalar fields may also take 2^n -ons as their values. We also tolerate that operators and fields support multiple sign selections, such as the inversion of the real axis and the handedness of external vector products for their eigenvalues. 2^n -ons offer n sign selections and contain n independent imaginary base numbers. With $n > m$, the 2^n -ons act like 2^m -ons in their lower m dimensions. Further, the 2^n -ons contain several subspaces of 2^m -ons. We use smoothly curved manifolds that are crossed by curves which form trails of 2^n -on numbers and that are locally touched by tangent spaces that can be interpreted as 2^n -on number fields.

The implementation of the proposition leads to a story of manipulators and manipulated observables. The number waltz feature ($c=ab/a$) of the 2^n -ons that becomes a noticeable effect for $n>1$ seems to play a crucial role in our model. If this model applies to quantum physics, then it may reveal why special relativity exists and brings clearness in the different notions of time that exist in quantum physics. The curvature tolerated by the higher dimension “eigenvalues” of the current manipulator reveals how the mentioned influences can be implemented as fields. This includes gravitational fields as well as the other fields such as electromagnetic fields.

The story also tells why the 2^n -ons produce things that feature a Lorentzian metric rather than a Riemannian metric. This occurs via an infinitesimal number transformation. Together with the sign selections that we decided to tolerate, the number waltz ($c=ubu^*$, where $uu^*=1$ and $u=1+\Delta s$) implements the actions of spinors. These facts explain why physicists prefer to use complex numbers, Clifford algebra, Jordan algebra, Grassmann algebra, pinors and spinors instead of 2^n -ons.

However, implementing quantum physics in a complex Hilbert space hides these interesting features and diminishes the insight that 2^n -ons can reveal.

This e-paper contains no reference list. References to other documents are presented inline and are mostly put in the form of hyperlinks. A local [toolkit](#) contains a collection of tools that otherwise must be grasped from internet. Much of the contents of the toolkit are directly or indirectly obtained from Wikipedia.

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Logic

The set of [quantum logical propositions](#) is lattice isomorphic with the set of closed subspaces of an infinite dimensional separable [Hilbert space](#) \mathbf{H} . This isomorphism means that quantum logical propositions can be represented by closed subspaces of a Hilbert space. The inner products of that Hilbert space can be defined using numbers of a 2^n -on number field. Taking $n > 2$ for that purpose raises numeric problems with the closure of the subspaces. Traditional quantum logic does not include any axioms that treat dynamics and it does not treat the influences of physical fields. It only specifies stationary relations that are possible between physical items.

It is an elucidating experience to try to implement a quantum logical proposition in Hilbert space in order to test its truthfulness. We will provoke by introducing in this example physical fields as well as dynamics.

The example proposition is:

All items in universe influence each other's position.

It can be answered with either yes or no. So, it is a valid quantum logical proposition. Proving 'yes' is cumbersome, but the 'no' is hardly less difficult. It requires finding an item of which the position is not influenced by at least one of the other items.

The statement includes quantifiers (*position*) and operational elements (*influence*). The set of axioms of traditional quantum logic does not treat these. As we will see, the influence of the universe of propositions (*items*) will put particular restrictions to the extension of quantum logic into the realm of a dynamic logic. This restriction is manifested in the occurrence of [inertia](#).

Translated in physical terms it means that in contrast to a uniform movement the acceleration of an item will go together with the action of a field. Notice that we use the words "goes together with" instead of "generates".

Translated in logical terms the final conclusion of our project runs: "

The exchange of atomic predicates in a surrounding proposition must be done in well-ordered and controlled steps. Otherwise the community of propositions will influence the considered surrounding proposition. "

When nature's logic is put in axioms, then influences that correspond to physical fields must follow from the axioms. Together with the specification of the origin of dynamics this will then result in a dynamic version of quantum logic. This mechanism will become clear later when we treat inertia.

I assume that this category of logic does not yet exist in mathematics. There exists a version of dynamic operational quantum logic, but it does not cover or mention the effects of the representation of physical fields in logic and it does not specify the origin of dynamics.

Atomic propositions

Atomic propositions are statements that are either true or false and which cannot be broken down into other simpler propositions. When an atomic proposition concerns a property, then it may contain the value of that property. For example "The speed is 5." The identity or the category of the property is "speed". The value of the property is 5. Its dimension is "meter per

second”, but that is another atomic statement and it is a fixed statement. This information is part of the type definition of the category “speed”.

The atomic propositions that are not type definitions form a set with a particular lattice structure. In this set we only consider atomic propositions that are independent of all other atomic propositions. Several choices of such sets exist. A subset consisting of members of a chosen set may be [canonical conjugates](#) of members of another set.

In Hilbert space the type definitions of atomic propositions that concern numeric variables are represented by operators. The values of the properties in the atomic propositions correspond to the eigenvalues of the operators or they are expectation values. Expectation values are statistically determined via a probability characteristic that characterizes both the operator and a physical item. See [State function](#).

Type definitions

Type definitions are propositions that describe and categorize subjects without specifying their variable values. A type definition of a category of atomic propositions specifies the type of property that these propositions treat. If that category is “speed”, then the definition also contains the dimension (e.g. meters per second) and the allowed range of the potential values. When the type definition concerns a more complex object that can act as an individual the definition will be called an item type definition. Item type definitions use atomic proposition types. When that item cannot be broken into simpler objects that still can act as an individual, then the type definition is an elementary type definition. Elementary type definitions are constructed of type definitions of atomic propositions.

If the item is not an elementary type, then its type specification is a system or sub-system type definition. A (sub)-system type definition is constructed of elementary item type definitions and atomic proposition types.

The type definitions form a set with a different lattice structure. Its structure is isomorph with the structure of classical logic. The elementary types form (a rather small) subset of the whole set of type definitions. Elementary types appear to divide into two categories: bosons and fermions.

In Hilbert space no representation for item type definitions exists.

Items

The first problem that is raised by constructing the representation of proposition (*) is to determine what in this representation stands for an item. The simplest solution is to attach a subspace of the Hilbert space to the item. The corresponding proposition can be phrased as: “*This is the item*”. Something either belongs to the subspace or it is outside that subspace. Everything that can be attributed to the item can also be attributed to this subspace. Each of these propositions belongs to a hierarchy for which the mentioned proposition forms the top. In this way the universe of items can be represented by a set of mutual orthogonal subspaces of the Hilbert space. Rays that are spanned by a single vector and that are connected with a numeric value can be considered as atomic propositions. Subspaces spanned by such rays that are related to the same type of value can be considered as statements with a wider scope. The rays can be subspace of an items subspace. The configuration of the subspace that represents an item will change as a function of the parameter that measures the progression of the dynamic behavior of the item. It is possible that not only the values of the atomic propositions

change. The types of these atomic propositions may change as well. This happens for example with atomic types that are each other's canonical conjugate.

Multidimensional subspaces exist that do not represent an item. They can be considered as **vacuum**. We will assume that on the average the 'filled' and the vacuum subspaces are evenly distributed over a connected part of the Hilbert space. The phrase "evenly distributed" means that the distance between the representations of items makes sense. Here we do not mean the distance related to the norm of Hilbert vectors, but the coordinate related distance that will be introduced [later](#). "Vacuum" does not say that these subspaces are empty. It is rather an indication that the subspace does not represent a dynamical object. The same holds for the rays that represent atomic propositions.

A nice extra is the fact that the subspace can be moved around (rotated around the origin) in Hilbert space. In this way it may be possible to implement the dynamics of items. This moving around does not mean that the vectors are moved around. It means that at each step of the move the set of vectors that span the considered subspace is **redefined**. The redefinition corresponds to a redefinition of the corresponding proposition. Thus, redefinition and the laws that govern redefinition convert the static quantum logic into a dynamic version of quantum logic. It will be shown that physical fields play a significant role in this redefinition.

With his bra-ket notation Dirac has provided us with a marvelous symbolism for vectors and even for operators. He did not provide us with symbols for subspaces. However, it is easy to extend his symbolism and indicate a subspace with a set of vectors that spans that subspace. For example $\{|f_s\rangle\}_s$ indicates a set of element vectors $|f_s\rangle$ with enumerator s that span a closed subspace. This set identifies the subspace. Different sets may identify the same closed subspace.

It is sensible to have one vector inside the item's subspace that is considered as characteristic for the location of the representation of the item in Hilbert space. We reserve the name **locator** for this vector. When the item is redefined, that vector may be redefined as well. This characteristic vector can be used to obtain a precise location of the subspace in Hilbert space. The process via which the locator is determined depends on the requirements. The requirements may be set in relation to an operator. In that case the [state vector](#) may play the role of the locator. Two or more bosons can share the same locator. Fermions that possess the same property values cannot share the same vector as a locator.

Atomic propositions are not considered to be statements that describe a physical item. The statement "This is the item" forms the top of a hierarchy of statements that all say something about the item. The hierarchy contains statements that define the item's type. Other members of the hierarchy specify the items constituents. Still other statements concern the item's atomic variables that together with the type definition specify the item's identity. For atoms the variables of the subsystems are hidden.

Physical fields are not physical items. For each physical field, every member of an orthonormal base of the Hilbert space corresponds to a value of the field. Physical fields have much in common with [wave functions](#). In quantum field dynamics the fields play a similar role as wave functions do in quantum mechanics. Each elementary type corresponds to a kind of physical field. With each fermion type an anti-type exists.

States

Where a unique closed subspace *represents* a given physical item, its state *characterizes* that item.

Function in Hilbert space

Every Hilbert vector $|f\rangle$ can be combined with the eigenvectors $\{|q\rangle\}_q$ of a normal operator Q to give a corresponding function $f(q)$. The values of this function follow from the inner products of the vector with the eigenvectors of the operator, while the corresponding eigenvalues $\{q\}$ form the variable of the function.

$$f(q) = \langle f|q \rangle$$

State

Where a unique closed subspace *represents* a given physical item, its state *characterizes* that item. In quantum physics, a quantum state is a set of mathematical variables that as far as is possible describes the corresponding physical item. For example, the set of 4 numbers $\{n, l, m_l, m_s\}$ define part of the state of an electron within a hydrogen atom and are known as the electron's quantum numbers. The observables that determine the state are mutually compatible. The position of the electron within the atom is a hidden property. If two operators are each other's canonical conjugate, then only one of them can participate in the state.

Quantum states can be either pure or mixed. **Pure states** cannot be described as a mixture of others. **Mixed states** correspond to a random process that blends pure states together. Realizations of elementary types are characterized by pure states.

When performing an observation on a quantum state, the result is generally described by a probability distribution, and the form that this distribution takes is completely determined by the quantum state and the operators that are related to the observation of the quantum state. The result of an observation is only determined probabilistically. In relation to the observables that determine the state a pure state is characterized by [a single Hilbert vector](#) and that vector corresponds in relation to these observables to a mathematical object known as a [state function](#). If a pure state corresponds with an eigenvector of the operator that represents the observation, then the result of the observation equals the corresponding eigenvalue. The probabilistic nature of observations reflects a core difference between classical and quantum physics.

Linear combinations (superpositions) of states can describe interference phenomena. A mixed state cannot be characterized by a single Hilbert vector. Instead, it is described by its associated [density operator](#) and it is represented by a closed Hilbert subspace.

State function

There exists a notion of [state](#) attached to each physical item. This state is often identified with a state function, which also may get the name **wave function**.

If the state is characterized by a set of independent properties, then each of these properties corresponds with an operator. The operators that together determine the state are mutually compatible. The wave function is the product of the probability amplitudes that correspond to the separate operators. The wave function corresponds to a Hilbert vector. This **state vector** is a type of characteristic vector that represents the probability amplitudes of a set of mutually compatible observables that correspond to the normal operators that determine the state.

The squared modulus of the probability amplitude is the probability density. The state vector as well as the state function can also be a function of time. Position can be a state characterizing observable. However, spacetime does not occur as an eigenvalue of an operator. The operators may vary. For example an operator may be replaced by its canonical conjugate. In that case, care must be taken that the operators that form the changed state are still compatible. Thus, even with the same physical item, the state function is not unique.

For the operator Q with eigenfunctions $|q\rangle$ and eigenvalues q the probability amplitude function $\psi(q)$ is given by

$$\psi(q) = \langle \psi | q \rangle$$

When Q is a [compact normal operator](#) then $\psi(q)$ is a continuous function. Then $\psi(q)$ has a Fourier transform $\varphi(p)$, where the operator P with eigenvectors $|p\rangle$ and eigenvalues p is the canonical conjugate of Q . Like $\psi(q)$, the function $\varphi(p)$ is also a function that characterizes the corresponding item and $|p\rangle$ is a characterizing vector. The parameters q and p may be quaternionic.

$$\varphi(p) = \langle \varphi | p \rangle$$

[Gleason's theorem](#) states that a probability measure $\mu(P)$ on the lattice $L(\mathbf{H})$ of projections P on closed subspaces of a Hilbert space \mathbf{H} corresponds to a non-negative Hermitian operator ρ with trace 1, such that $\mu(P) = \text{tr}(\rho P)$. When the projections P_q correspond to the rays formed by the eigenvectors $|q\rangle$ of operator Q and $\mu_i(P_q)$ corresponds to the considered physical item, then $\mu_i(P_q) = \langle q, \rho_i q \rangle$ corresponds to the square of the modulus of the wave function $\psi_i(q)$. ρ_i is the [probability density operator](#) corresponding to μ_i . The probability measure μ is a regular function. $\mu_i(P_q)$ is zero outside the subspace that represents the considered physical item.

With respect to the correspondence with traditional quantum logic, it is wrong to take any characteristic vector including the locator or any function including the wave function as the *representative* for the item. It is ridiculous to expect that a single vector carries all properties of a complex physical item, such as a DNA molecule or an elephant. The Heisenberg uncertainty relation also offers objections against this single vector based representation.

In quantum mechanics the wave function can be interpreted as the combination of a stationary vector and a progression operator. The progression operator has the form $A \cdot \exp(S/\hbar)$. A is Hermitian and positive. S is anti-Hermitian. This is reflected in the Hamilton-Jacobi equation.

In quantum field theory the fields are replacing the wave function. Thus a field may be interpreted as the amplitude of the probability to find something at the location of the field value. For bosons that something may be interpreted as a virtual particle. For fermions that something may be interpreted as a pair of virtual particles. Each type of virtual particle has its own type of field.

Probability density

The [probability density function](#) $p(q) = |\psi(q)|^2$ of an absolutely continuous random variable q is a function that describes the relative chance for this random variable to occur at a given point in the Q observation space. The probability for a random variable to fall within a given set is given by the integral of its density over the set.

The probability [density operator](#) ρ is positive-semi-definite ($\forall |f\rangle \in \mathbf{H} \{ \langle f | \rho | f \rangle \geq 0 \}$), self-adjoint ($\rho = \rho^\dagger$), and has trace one ($\text{tr}(\rho) = 1$). For the operator Q with eigenfunctions $|q\rangle$ and eigenvalues q with probability amplitude $\psi(q)$, the density operator ρ is given by

$$\rho = \sum_q \{ |\langle \psi | q \rangle|^2 \cdot |q\rangle \langle q| \}$$

[Von Neumann entropy](#) is defined using the density operator of physical items.

The operator A can be decomposed

$$A = \sum_a |a\rangle \langle a|$$

For the state $|\psi\rangle$ the **expectation value** $\langle A \rangle$ for the observable A is

$$\langle A \rangle \equiv \langle \psi | A | \psi \rangle = \sum_q \{ |\langle \psi | q \rangle|^2 \cdot \langle q | A | q \rangle \} = \text{tr}(\rho A)$$

Observables

In Hilbert space observables are represented by operators. The observed value is represented by an eigenvalue or by the expectation value of the operator that represents the observable.

Numbers

The Hilbert space can be specified by using a number field that allows the mutual orthogonalization and the closure of subspaces. The real's, the complex numbers and the quaternions can perform that job. Horwitz showed that even the octonions with some trouble can achieve this (see: <http://arxiv.org/abs/quant-ph/9602001>). The real's, the complex numbers, the quaternions and the octonions are the only normed division algebras and they are the only alternative division algebras. In general the octonions are not associative, but the product of two octonions that belong to the same quaternionic subfield is associative. Neither all quaternions nor all octonions commute. However, within complex subspaces the numbers commute.

We will take the following freedom. The fact that a given number field is used for specifying linear combinations of Hilbert space vectors does not mean that eigenvalues of operators must also be restricted to that same number field. In this sense a Hilbert space specified over the quaternions may allow eigenvalues of operators that are taken from the octonions or even higher 2^n -ons (see <http://www.math.temple.edu/~wds/homepage/nce2.pdf> or the [toolkit](#)). The problem with higher dimension 2^n -ons is that their number characteristics deteriorate with n . However, as long as the (full) eigenvalues are not used to construct linear combinations of vectors, or to specify the inner products of the Hilbert space, there is no problem. All higher dimensional 2^n -ons contain several subspaces that are lower dimensional 2^m -on number fields. Further, 2^n -ons behave like 2^m -ons in their lower 2^m dimensions.

In general the elements of curves or curved manifolds are themselves not numbers. So, they cannot be used as eigenvalues. Smoothly curved trails of objects that locally resemble 2^n -ons can be treated with the Frenet-Serret frame toolkit. Number fields can be **attached** as tangent spaces to smoothly curved manifolds. In that way the elements of the curves and the manifolds obtain number characteristics in a small enough environment. Sequences or sets of operators can locally have eigenvalues that are numbers which can be considered as member of smooth curves or of the tangent space of a curved manifold at that location. In that way the elements of smooth curves or of curved manifolds can be related to the corresponding

eigenvalues. 2^n -ons are ideally suited for this purpose. This means that the eigenspaces of the subsequent operators in a trail need not overlap. These eigenspaces are only used locally. When curvature and bending of the operator trail diminish, the dimension of the local number field can be lower. When the curvature and the bending increase, the dimension must be higher. This will be reflected in the dimensionality of the local eigenvalues. Apart from the application as eigenvalues of operators the 2^n -ons are suited as values of physical fields.

We will restrict to the 2^n -ons as extensions of the quaternions. As we stated, the numbers created with the Cayley-Dickson construction are not so well behaved. Alternatives are the use of Clifford algebras, Jordan algebras or Grassmann algebras. We will show that in the Hilbert space the 2^n -ons for $n > 1$ automatically introduce these latter algebras through their number waltz.

The **niners** are the most extensive numbers that still keep a reasonable set of number characteristics. More precisely said the 2^n -ons, even those that have a higher dimension than the octonions, keep reasonable number characteristics in the space spanned by their coordinates that have an index lower than nine. The real numbers, the complex numbers, the quaternions and the octonions completely fall within these boundaries. The above hyperlink describes exactly what characteristics the niners retain.

The subspace of the 2^n -on field that is spanned by the first 2^m dimensions acts as a 2^m -on number field. Thus in a dynamic situation, an octonionic operator acts locally as a quaternionic operator. In a smaller or more flat region it acts as a complex operator and at “nano”-locality as a real (or better as an imaginary) operator.

2ⁿ-on construction

The 2^n -ons use the following doubling formula

$$(a, b) (c, d) = (a \cdot c - (b \cdot d^*)^*, (b^* \cdot c^*)^* + (b^* \cdot (a^* \cdot ((b^{-1})^* \cdot d^*)^*)^*)^*) \quad (1)$$

Up until the 16-ons the formula can be simplified to

$$(a, b) (c, d) = (a \cdot c - b \cdot d^*, c \cdot b + (a^* \cdot b^{-1}) \cdot (b \cdot d)) \quad (2)$$

Up to the octonions the Cayley Dickson construction delivers the same as the 2^n -on construction. From $n > 3$ the 2^n -ons are ‘nicer’ than the Cayley Dickson numbers. They keep more useful number characteristics. The 2^{n+1} -ons contain the 2^n -ons as the sub-algebra of elements of the form $(a, 0)$

Waltz details

The 16-ons lose the continuity of the map $x \Rightarrow xy$. Also, in general holds $xy \cdot x \neq x \cdot yx$ for 16-ons. However, for all 2^n -ons the base numbers fulfill $e_i e_j \cdot e_i = e_i \cdot e_j e_i$. All 2^n -ons feature a conjugate and an inverse. The inverse only exists for non-zero numbers. The 2^n -ons support the **number waltz**

$$c = a \cdot b a^{-1}. \quad (3)$$

Often the number waltz appears as a unitary number waltz

$$c = u^* \cdot bu \quad (4)$$

where u is a unit size number and u^* is its conjugate $u \cdot u^* = 1$.

In quaternion space the **quaternion waltz** $a \cdot b \cdot a^{-1}$ can be written as

$$a \cdot b \cdot a^{-1} = \exp(2 \cdot \pi \cdot \underline{\mathbf{i}} \cdot \varphi) \cdot b \cdot \exp(-2 \cdot \pi \cdot \underline{\mathbf{i}} \cdot \varphi) \quad (5)$$

$$= b - \underline{\mathbf{b}}_{\perp} + \exp(2 \cdot \pi \cdot \underline{\mathbf{i}} \cdot \varphi) \cdot \underline{\mathbf{b}}_{\perp} \cdot \exp(-2 \cdot \pi \cdot \underline{\mathbf{i}} \cdot \varphi)$$

$$= b - \underline{\mathbf{b}}_{\perp} + \exp(4 \cdot \pi \cdot \underline{\mathbf{i}} \cdot \varphi) \cdot \underline{\mathbf{b}}_{\perp}$$

$$\Delta b = (\exp(4 \cdot \pi \cdot \underline{\mathbf{i}} \cdot \varphi) - 1) \cdot \underline{\mathbf{b}}_{\perp} \quad (6)$$

$$= (\cos(4 \cdot \pi \cdot \varphi) + \underline{\mathbf{i}} \cdot \sin(4 \cdot \pi \cdot \varphi) - 1) \cdot \underline{\mathbf{b}}_{\perp}$$

$$= \exp(2 \cdot \pi \cdot \underline{\mathbf{i}} \cdot \varphi) \cdot 2 \cdot \underline{\mathbf{i}} \cdot \sin(2 \cdot \pi \cdot \varphi) \cdot \underline{\mathbf{b}}_{\perp}$$

$$\|\Delta b\| = \|2 \cdot \sin(2 \cdot \pi \cdot \varphi) \cdot \underline{\mathbf{b}}_{\perp}\| \quad (7)$$

Another way of specifying the difference is:

$$\Delta b = (a \cdot b - b \cdot a) / a = 2 \cdot (\underline{\mathbf{a}} \times \underline{\mathbf{b}}) / a \quad (8)$$

$$\|\Delta b\| = 2 \cdot \|\underline{\mathbf{a}} \times \underline{\mathbf{b}}\| / \|a\| \quad (9)$$

Infinitesimal number transformation

The number v is close to 1. Thus $v = 1 + \Delta s$. Let us investigate the transform $c = v^* \cdot b \cdot v$.

$$c = (1 + \Delta s^*) \cdot b \cdot (1 + \Delta s) \quad (10)$$

$$= b + \Delta s^* \cdot b + b \cdot \Delta s + \Delta s^* \cdot b \cdot \Delta s$$

$$\approx b + \Delta s^* \cdot b + b \cdot \Delta s$$

$$= b + \Delta s_0 \cdot b + 2 \cdot \underline{\mathbf{b}} \times \Delta \mathbf{s}$$

$$\Delta b = \Delta s_0 \cdot b + 2 \cdot \underline{\mathbf{b}} \times \Delta \mathbf{s} \quad (11)$$

This comes close to the effect of an infinitesimal number waltz, especially when $\Delta s_0 = 0$. In that case $\Delta b_0 = 0$ and $\Delta \mathbf{b}$ is perpendicular to $\Delta \mathbf{s}$.

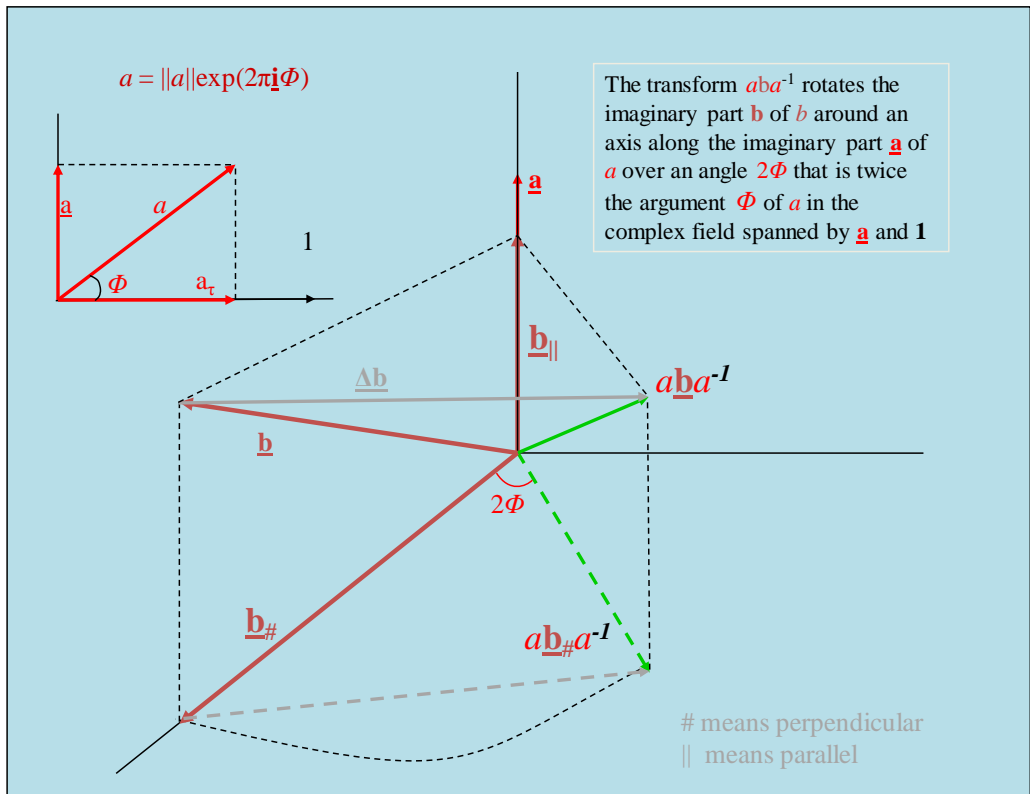


Figure 1. The rotation of a quaternion by a second quaternion.

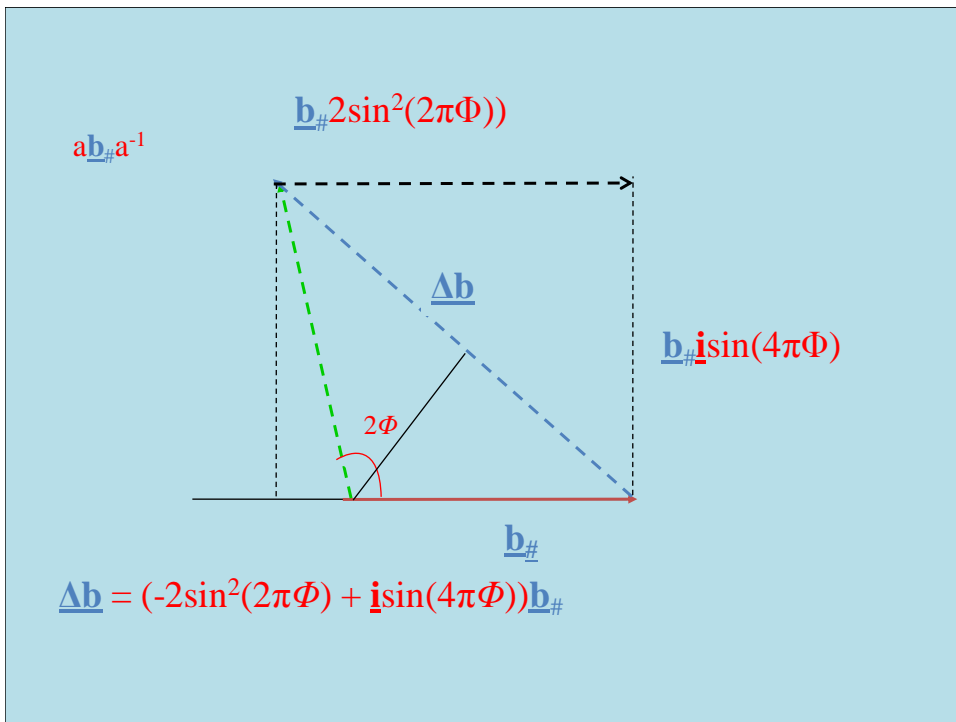


Figure 2: The difference after rotation

For 2^n -ons with $n > 1$, $a \cdot ba^{-1}$ in general does not equal b . This effect stays unnoticed when quantum mechanics sticks to a complex Hilbert space.

Sign selections

The paper that describes 2^n -ons does not describe the choice for right or left handedness of the external vector product. So, we do it here. The generally accepted convention is to let the

handedness depend on the orientation of the underlying \mathbb{R}^n space. However, when numbers are constructed via the Cayley-Dickson construction or the 2^n -on construction then the handedness follows from the applied construction formula. We want to get rid of these restrictions, because we want to give operators the freedom to select the handedness and other sign selections of their eigenvalues.

The 2^n -ons have n independent binary base numbers and n sign selections. The real numbers do not offer a sign selection. The complex numbers offer the selection of the sign of the real axis. This is inherited by all higher 2^n -ons. The quaternions have two independent imaginary base numbers and offer an extra sign selection that represents the handedness of its external product. The octonions have three independent imaginary base numbers and offer an extra sign selection for the handedness in external products that involve this new base number.

Need for spinors

In the number waltz the current manipulator only needs an argument α in order to turn the subject over 2α . This is typical behavior for spinors. Spinors also have a storage place for the handedness of rotations. By using the number waltz and the sign selections the 2^n -ons can perform the same act as the spinors. Spinors are only required when quantum mechanics is restricted to complex Hilbert spaces. Spinors are the carriers of the spin phenomenon. Thus in our model the sign selections form in combination with the number waltz the carriers of the spin. Because a strange trick is played with the real parts of eigenvalues, the influence of the selection of the sign of the real axis will be revealed [later](#).

The approach taken in this paper might cause a revival of the importance of the hyper complex numbers that turned in oblivion when Gibbs introduced his vector analysis.

The GPS of the Hilbert space

We already noticed that there exists a requirement for a notion of distance both in quantum logic as well as in Hilbert space. The inner product of the Hilbert space specifies a norm and on its turn the norm offers the definition of the notion of distance between Hilbert vectors. In terms of this distance all vectors in an orthonormal base have the same inter-distance, which equals $\sqrt{2}$. For that reason we will introduce here another notion of distance, the **coordinate related distance**. From now on, when in this paper the word distance is used in relation with Hilbert spaces, then the notion of coordinate related distance is meant, rather than the norm related notion. By assigning a characteristic [locator](#) vector to each closed subspace it becomes possible to obtain a precise coordinate related distance between subspaces. The distance and the locator can be transferred to corresponding objects in quantum logic.

The next step is the introduction of a suitable GPS system in Hilbert space. This can be done by taking an orthonormal base of Hilbert vectors and add 2^n -on values to them. Let us take the rational quaternions as an example. This construction defines a normal operator Q with countable infinite number of eigenvectors $|q\rangle$ and corresponding eigenvalues q . We will use the name **coordinate space** for the eigenspace of the **coordinate operator** Q .

When we speak about the coordinate distance between two vectors $|f\rangle$ and $|g\rangle$ in Hilbert space, then we mean the distance between the values of $\langle f|Q|f\rangle$ and $\langle g|Q|g\rangle$.

Next take the polar decomposition of Q in a unitary part U and a positive operator N . The eigenspace of U is the **uni-coordinate space**. Like the unit sphere of the Hilbert space, the uni-coordinate space is an affine space. Besides of that there also does not exist a preferred direction and there does not exist a preferred scale. An infinite number of eigenvectors of Q correspond to each separate eigenvalue q of U .

Position

The original proposition (*) speaks about the position of the item. The position must be related to something that is available in the Hilbert space. The Hilbert space is defined over a number field. Thus we might attach a number of this field (or a higher $2n$ -on) to the subspace that represents the item. That number must represent position. The natural way of attaching numbers to subspaces of a Hilbert space is via the concept of eigenvalues of normal operators. Any symmetry transform of the coordinate operator Q meets the requirements. However, the separable Hilbert space has a countable dimension. It means that the eigenvalues may offer a dense coverage of a connected part of the number field, but it is not a closed coverage. It does not include all limits of all convergent rows. Thus it is sensible to attach a tiny environment of the actual eigenvalue to each eigenvector. In this way the position is expressed in a tiny environment rather than in a single number. At least the position is represented by a single eigenvector and in this way the whole number field is covered by the set of eigenvectors. The eigenvector represents an atomic proposition that represents the position attribute of the considered item. The eigenvector lies inside the subspace that represents the item. The corresponding atomic proposition states that the position of the item lies inside the environment that is represented by the eigenvector.

Now the position is connected to eigenvectors. The physical item is connected to a subspace rather than to a single vector. So we can use the localizer as a more precise indicator of the position of the physical item. On the other hand physical items are characterized by wave functions. Wave functions represent a probability amplitude. This probability can be the probability that after measuring the position the argument of the function is found as the result. In that case the wave function indicates the probability of finding the position of the localizer.

The fact, that the operator must be bounded in order to guarantee that its eigenvectors span the whole Hilbert space, is not crucial to our model. It is sufficient when all positions that are connected to the items stay in a finite sphere.

The deliberations that the eigenvalues of operators need not stay in the number field that is used to specify the Hilbert space also hold for the position operator. In this way, the positions may be elements of a curved manifold. In this case we will call the position operator an extended position operator. The curved position space may be seen as the result of the actions of the fields. The fields themselves can be seen of functions of the observed curved space or as a function of the untransformed position.

The origin of dynamics

If we want to discover the origin of dynamics, we must first determine what the static structure of nature is. We already found one ingredient of this skeleton: the lattice structure of quantum logic and the corresponding lattice structure of the closed subspaces of a Hilbert space. Both structures are only defined in a static way. Nothing is said about their dynamics. Besides of these static relations the concept of wave functions and density operators offer insight in the probability and information content of these relations.

This chapter investigates the next component of the static structure of nature: the static structure of the influences. First we analyze the obvious dynamic properties of these influences. Next we analyze their static structure. Finally we try to find the origin of dynamics.

Influence

The original proposition (*) talks about influencing the position of an item. This implies that the position of the item changes due to the mentioned influence. Thus when the influence occurs, the eigenvector that represents the position of the item is exchanged against another eigenvector. That other eigenvector corresponds to another environment inside the eigenspace of the position operator. The new eigenvector takes the role of the old eigenvector and is the new characteristic for the item's position. This replacement may take place inside the subspace, which represents the considered item, or the original eigenvector moves outside the subspace, while the new eigenvector moves in or stays in the subspace. In both cases the eigenvectors of the position operator move with respect to the vector that characterizes the subspace of the item. The movement is relative and takes place inside the Hilbert space. Another possibility is that the eigenvectors stay, but the corresponding eigenvalues change while the Hilbert subspace moves.

Thus, there is a way to implement influence in Hilbert space. The influence causes a move of the item's subspace relative to one or more eigenvectors of the position operator. The original proposition (*) claims that this movement is caused by other items. We must check whether this is true.

If this is true then influences are the motor behind the dynamics of the items.

The universe of items

The original proposition(*) states that all items influence each other's position. This includes that all items influence the considered item. Part of the items compensates each other's influences on the currently considered item. It will be shown that this holds for the largest part.

Inertia

The influence may decrease with distance according to some function $f(r)$ of the distance r . However the number of contributing items increases with the distance. The most probable result is that the strongest influence comes from the cooperative activity of the most distant items. Any variation of the influences of the distant items averages away. This also holds for the distribution density of the items. So there exists a fairly uniform background influence caused by the universe of items. What happens can be deduced from an equivalent of Denis Sciama's analysis. We will take his analysis as a guide. Sciama's analysis uses a different setting: the (observed) 3D space and coordinate time. (See: <http://arxiv.org/abs/physics/0609026v4.pdf>). The critique on his approach is that he uses instantaneous action on large distances, which in his setting is in conflict with special relativity. In this setting we do not (yet) encounter special relativity. We use the coordinate space and the progression parameter as our setting. Locations in coordinate space represent locations in Hilbert space.

The most important aspects of the analysis are:

The total potential at the location of the influenced subject is

$$\Phi = - \int_V \frac{\rho}{r} dV = -\rho \int_V \frac{dV}{r} \quad (13)$$

(See: http://en.wikipedia.org/wiki/Newtonian_potential) The integral is taken over the coordinate space V . Thus, in fact the integral is taken over the unit sphere of Hilbert space. r is the distance from the actor to the location of the subject. The considered subject is located somewhere in the affine coordinate space. All other subjects have positions relative to that considered subject. At large distances, the density ρ of the contributing items can be considered to be uniformly distributed. Also any variance in strength other than the dependence on r becomes negligible. We take the average of the strength as the significant parameter. We combine it with ρ . Therefore the average of ρ can be taken out of the integral. Thus, apart from its dependence on the average value of ρ , Φ is a huge constant. (Sciama relates Φ to the gravitational constant). We can consider the universe as a very large rigid body. If nothing else happens then all influences compensate each other.

If the subject moves relative to the universe with a uniform speed \mathbf{v} , then a vector potential \mathbf{A} is generated.

$$\mathbf{A} = - \int_V \frac{\mathbf{v} \cdot \rho}{c \cdot r} dV \quad (14)$$

Both ρ and \mathbf{v} are independent of r . Together with the constant c they can be taken out of the integral. Thus

$$\mathbf{A} = \Phi \cdot \mathbf{v} / c \quad (14)$$

The two volume integrals concern the two components of a simple case of the [Helmholtz's decomposition theorem](#). $\mathbf{F}(\mathbf{r})$ is a vector point function that vanishes faster than $1/r^2$ at infinity. In a **static** situation hold:

$$\nabla \cdot \mathbf{F}(\mathbf{r}) = 4 \cdot \pi \cdot Q(\mathbf{r}) \quad (15)$$

$$\nabla \times \mathbf{F}(\mathbf{r}) = 4 \cdot \pi \cdot \mathbf{I}(\mathbf{r}) \quad (16)$$

$$\mathbf{F}(\mathbf{r}) = \nabla^2 \mathbf{A}(\mathbf{r}) = \mathbf{F}_1(\mathbf{r}) + \mathbf{F}_2(\mathbf{r}) \quad (17)$$

$$\mathbf{F}_1(\mathbf{r}) = \nabla \Phi(\mathbf{r}) \quad (18)$$

$$\mathbf{F}_2(\mathbf{r}) = -\nabla \times \mathbf{A}(\mathbf{r}) \quad (19)$$

$$\nabla \times \mathbf{F}_1(\mathbf{r}) = 0 \quad (20)$$

$$\nabla \mathbf{F}_2(\mathbf{r}) = 0 \quad (21)$$

The notions of charge and current correspond to equivalent notions in [Noether's theorem](#). Thus charge may symbolize mass.

These formulas translate directly into formulas for electro statics and magneto statics. They are taken from “Mathematical Handbook for Scientists and Engineers”; G.A. Korn and T.M. Korn; McGraw-Hill;1968; section 5.7. Here we talk about inertia.

$Q(\mathbf{r})$ represents the local charge density ρ . $\mathbf{I}(\mathbf{r})$ stands for the current density represented by the product of \mathbf{v} and ρ . The speed is measured in the coordinate space, but this represents a uniform movement in Hilbert space. Here the trail progression parameter plays the role of manipulator time. (Be aware, this is not our usual notion of time).

The Helmholtz decomposition theorem only concerns the static versions of the fields. It is related to the fact that the Fourier transform of a vector field can be split in a longitudinal and a transversal version. There also exists a corresponding split of the multi-dimensional Dirac delta function in a longitudinal and a transversal version. In curved manifolds the Helmholtz decomposition should be replaced by the Hodge decomposition. It is remarkable that in Helmholtz decomposition only two perpendicular directions are treated and not three!

A variation of \mathbf{v} goes together with a variation of A . On its turn this goes together with a non-zero field \mathbf{g} which is the **dynamic** version of F_1 . The path of the item in the coordinate space maps to a path on the manifold in action space.

Sciama uses a Maxwell equation to explain the relation between $\partial\mathbf{v}/\partial t$ and \mathbf{g} . Our setting differs, but the main reasoning is the same.

$$\mathbf{g} = -\nabla\Phi - \frac{1}{c} \cdot \frac{\partial A}{\partial t} \tag{22}$$

Remark: Compare this dynamic version with the static version (18). As soon as we turn to the dynamic version (22) an extra component of field \mathbf{g} appears that corresponds to acceleration $\partial\mathbf{v}/\partial t$. (See for derivation of Maxwell equations e.g. the online book <http://www.plasma.uu.se/CED/Book>; formula 3.25)

$\nabla\Phi$ is negligible. This means that if the influenced subject moves with uniform speed \mathbf{v} , then $\mathbf{g} \approx 0$. However, any acceleration goes together with a non-zero field. In this way the universe of items causes inertia. The field \mathbf{g} appears as part of the action in the eigenvalue of the manipulator that currently characterizes the accelerating item.

We have used the coordinate space as a playground to implement an equivalent of Sciama’s analysis. The analysis uses the fact that every item in universe causes an influence and that this influence reduces according to $f = -k/r$. (Compare this with Bertrand’s theorem in Wikipedia)

A uniform movement in Hilbert space does not generate a reaction of the universe of items. Any alteration will cause as a reaction **a field that will appear in the eigenvalue of the current manipulator trail element**. It appears in the argument of that eigenvalue. The physical name for it is **action**. It usually gets the symbol **S**. When the trail of eigenvalues coincides with a geodesic, then it can be travelled field free.

It must be noticed that the original analysis of Sciama uses observable position space rather than Hilbert space, coordinate space or action space and it uses a different notion of time. However, the general conclusion stays the same. Sciama’s analysis is criticized because it

uses a infinite speed of information transfer. Since we do not work in observable position space, we do not encounter coordinate time. So for us, this criticism is misplaced. ([Coordinate time](#) is related to observations of position.)

The path in the coordinate space corresponds with the trail in Hilbert space. This on its turn corresponds with a trail in manipulator space and a trail in action space. Let us consider a trail in action space with unit speed. Now we can apply the toolkit of the Frenet-Serret frames technology in order to [analyze this trail](#). After re-parameterization such that t also represents the traveled distance, the trail becomes a geodesic.

Nearby items

Items that are located nearby have a different effect. In general their influence will not have an average strength. Further these items are not uniformly distributed. Still their influence depends on inter-distance as $f = -k/r$. As a consequence their influences form a landscape of which the effects will become sensible in the action of the current manipulator of the considered item. This landscape will form a curved action space. The considered item will try to follow a geodesic through that curved space.

Rotational inertia

Besides linear inertia there exists rotational inertia. In a non-rotating universe hold near the origin $\mathbf{A} = 0$ and $\Phi = -c^2/G$. We choose units such that $c=G=1$. In a universe rotating slowly with angular speed ω hold

$$\begin{aligned} A_x &= \omega \cdot y \\ A_y &= -\omega \cdot x \\ A_z &= 0 \\ \Phi &= -\sqrt{1 + (\omega \cdot r)^2} \end{aligned}$$

A constant angular movement meets the fields that correspond to a centripetal force.

The field \mathbf{E} has the form

$$\mathbf{E} = \frac{\omega^2 \mathbf{r}}{\sqrt{1 + \omega^2 r^2}} \quad (23)$$

An added uniform speed \mathbf{v} meets the fields corresponding to a Coriolis force.

$$\mathbf{H} = \nabla \times \mathbf{A} = 2 \cdot \boldsymbol{\omega} \quad (24)$$

$$\mathbf{v} \times \mathbf{H} = 2 \cdot \mathbf{v} \times \boldsymbol{\omega}$$

The forces are usually considered as fictitious but they are actually caused by inertia. Sciama treats them in section 5 of his paper. Like fields of linear inertia these rotation related fields correspond to actions that will have their place in the argument of the manipulator.

Interpretation in logical terms

The results of the analysis mean that when the redefinition of the set of vectors that belongs to the representation of the item occurs such that this corresponds to a uniform movement, then the influences of the universe of items compensate each other. Otherwise, the universe of

items reacts with a corresponding field. That field manifests as an action of the current manipulator. Local variance in the distribution of items causes a variation in the influences.

It seems that quantum logic and Helmholtz decomposition together define an important part of the static relations that exist in physics. The fields appear to resist the disturbance of the interrelations in the lattice of quantum propositions. In dynamical sense this lattice might step from one static status quo to the next. After a step new conditions are established that again fulfill the laws that govern the static situation. If this is a proper interpretation, then it is likely that the progression step is taken universe wide. After each step the positions of the physical items relative to the fields have changed, thus when the fields are not uniformly distributed, the items meet a different field configuration. The next step is taken with these new conditions.

Quantum logic only defines a static skeleton in which the dynamics of quantum physics takes place. To make it a dynamic logic the set of axioms must be extended. The new axioms must state that all propositions influence each other. The influence depends on their mutual distance. In stationary conditions, which include uniform motion, these influences compensate each other. When an atomic proposition that concerns an element of an ordered set is replaced in a non-ordered fashion, meaning that the distance between the replaced elements does not stay the same, then the universe of all propositions will react such that the influences of the other propositions no longer compensate each other. The disordered influences counteract the disordered replacement. Besides of that the local variance in the distribution of items also cause a variation in the influences that propositions have with respect to each other.

In Hilbert space these influences are implemented in the actions of manipulators. In quantum physics the influence appears as a set of fields.

Schrödinger or Heisenberg picture

For global rotations around its origin the Hilbert unit sphere acts as an affine space. It does not matter whether the eigenvectors of operators or the subspace that represents the item is moved. We can take the picture in which the subspace stays fixed, while the vectors move.

This is the **Heisenberg picture**.

We can also take the picture in which the vectors stay fixed and the subspace moves. This is the **Schrödinger picture**.

We are only interested in the consequences. These are determined by the relative movement, not by the absolute movement.

Unitary transform

A unitary transform is a bounded normal operator. It has unit sized eigenvalues and to each of these eigenvalues correspond one or more eigenvectors that are mutually orthogonal. Unitary transforms keep the value of inner products untouched. Unitary transforms are completely determined by their vector replacement characteristics, their eigenvectors and the corresponding eigenvalues. An extra characteristic is for example the smoothness of their eigenspace. A Fourier transform is an example of a unitary transform.

When a unitary transformation U is applied to an arbitrary vector $|f\rangle$, which is not an eigenvector, then that vector is transferred into another vector $|g\rangle = |U f\rangle$, which has the same norm. If $|f\rangle$ is an eigenvector of U then $|f\rangle$ is **not** transferred to a different vector, but it is multiplied with the corresponding eigenvalue. Also in this case the norm stays the same.

If a unitary transform is applied to two vectors and one is an eigenvector and the other is not an eigenvector, then the inner product stays the same. The non-eigenvector turns around the foot of the eigenvector, but keeps its angle.

In general, the transfer of a closed subspace requires a set of parallel unitary transforms. If we take a set of vectors $\{|f_s\rangle\}_s$ that together span a closed subspace, then a set of suitable unitary transforms $\{U_s\}_s$, can in parallel transfer all vectors of this set such that after the transform $|g_s\rangle = |U_s f_s\rangle$ the set $\{|g_s\rangle\}_s$ spans the new subspace.

When a unitary operator U is applied to the eigenvector $|q\rangle$ of an operator Q with eigenvalue q , then the eigenvector is transferred into another vector $|U q\rangle$. In general $|U q\rangle$ is not another eigenvector of Q . The expectation value for $|QU q\rangle$ is no longer q , but

$$\langle q | U | QU q \rangle = \langle q | U^\dagger QU q \rangle \quad (12)$$

Or, with other words the operator Q is redefined to $U^\dagger QU$.

The sketched situation can be refined for any instant t occurring after $t=0$. We can treat it more generally by chopping the path from $\{|f_s\rangle\}_s$ to $\{|g_{st}\rangle\}_s$ into a **trail** of infinitesimal steps of size Δt that is achieved by a set of infinitesimal transforms $\{U_{st}\}_{st}$, where $|g_{st}\rangle = |\prod U_{st} f_s\rangle$ and $U_{st} \approx 1 + \Delta S_{st}$. t acts as the trail progression parameter. It is not identical with our notion of time. The infinitesimal transforms U_{st} work in parallel as well as in sequence. ΔS_{st} represents the current local action step.

As indicated above the trail $\{U_{st}\}_{st}$ causes a redefinition of the operators that have eigenvectors in the considered subspace. If we do not want to redefine operators, then we must redefine subspaces. In that case we must NOT use unitary transforms.

The Heisenberg picture conforms to the description with unitary transforms where operators are redefined. When this is done in small steps, then the redefined operator becomes a function of progression parameter t .

Redefinition

Let us suppose that there exists a dynamical equivalent of the traditional quantum logic. The equivalent of a move of a physical item in the lattice of propositions is a redefinition of a subset of the propositions. The redefinition occurs in terms of atomic propositions that describe the properties of the physical items. In the Hilbert space this corresponds with a redefinition of the Hilbert subspace in terms of the eigenvectors that belong to the new eigenvalues.

A thing that moves Hilbert subspaces without touching the eigenvectors of normal operators will be called a **redefiner**. The effect of the action of the redefiner must be similar to the effect of the trail of parallel unitary transforms treated in the previous paragraph.

To a large degree the redefiner must have similar properties as the trail of parallel infinitesimal unitary transformations treated above. The redefinition keeps the inner products of vectors intact and like a unitary transform rotates vectors around the origin of the Hilbert space, the redefiner rotates subspaces around the origin of Hilbert space. In contrast to a unitary transform it does not change the eigenvectors of normal operators. It leaves the

operators untouched. Like the trail of unitary transforms the redefinition works in infinitesimal steps. These infinitesimal actions also form a trail. In this way the manipulated subspace can move continuously through Hilbert space. The redefiner steps from one stationary situation to the next. The Schrödinger picture conforms to the description with a redefiner. The result for the position of the locator must be the same as it was under the influence of the set of parallel infinitesimal unitary operators in the Heisenberg picture. The redefiner moves the subspace such that the new locator position is similar to the value as was established by the redefined position operator. It means that during the redefiner step the position of the locator undergoes an infinitesimal number transform that is equivalent to the infinitesimal transform that is established by the redefined position operator. That redefinition was caused by the parallel infinitesimal unitary transforms. If this interpretation is correct, then it means that the expectation value of the moved eigenvector $|U q\rangle$ will be given by $\nu^* q\nu$ for number ν that corresponds to the current effective “eigenvalue” of the redefiner. The number $\nu = 1 + \Delta s$ is close to unity. Δs is the action step.

Trails

In fact the t step is the [redefinition](#) step. The subsequent replacement of vectors and the replacement of the corresponding eigenvalues can be interpreted as a rather continuous movement of the corresponding characteristic subjects. Here we encountered five different trails.

1. The trail of subsequent manipulators (infinitesimal unitary transforms or infinitesimal redefiners) that each perform an infinitesimal action.
2. The trail of eigenvectors $|q_t\rangle$ and subspaces, which with respect to the manipulators are characteristic for the considered item.
3. The trail of corresponding “eigenvalues” of the manipulator.
4. The trail of corresponding observables Q_t .
5. The trail of corresponding observed position values q_t .

The trail of eigenvalues q_t corresponds to a trail of eigenvectors $|q_t\rangle$. This, on its turn corresponds to a trail in coordinate space and a trail in Hilbert space.

Cycles

It is quite possible that subsequent steps are done in cycles of two or more steps. It is obvious that movements inside an item are cyclic. In ideal circumstances these movements are harmonic.

Redefiner

The concept of dynamic manipulator gives us reason to introduce a **new type of operator**: the redefiner \mathcal{R} . This operator moves subspaces, but leaves vectors untouched. It works in infinitesimal steps. It is easily interpreted as a function \mathcal{R}_t of the progression parameter t . Its scope spans the Hilbert space. The effect of each step on an item is similar to the effect of a set of parallel infinitesimal unitary transforms $\{U_{ts}\}_s$. The current “eigenvalue” is a number, which is close to unity. The redefiner accepts 2^n -ons as “eigenvalues”.

The redefiner has an equivalent in an extended quantum logic, where it redefines propositions that concern the same objects as are represented by the closed subspaces of the Hilbert space that are moved by \mathcal{R}_t . There seems to be no objection against the assumption that \mathcal{R}_t has a global scope. If we take that point of view, then the progression parameter t also has a global scope.

With this interpretation, the redefiner is a universe-wide stepper. It transforms the universe from one static situation to the next static situation. As we will see in the section on inertia, these static situations are governed both by traditional quantum logic and by the Helmholtz/Hodge decomposition theorems. After each step the status quo of subspaces and fields is reestablished. However, after the step the conditions have been changed. After each step the position of the physical item relative to the fields has changed, thus when the fields are not uniformly distributed, the item meets a different field configuration. This is the way that macroscopic dynamics takes place in quantum physics. (See the section on [optics](#))

The manipulator

In the Heisenberg picture the manipulator is represented by the trail of sets of infinitesimal unitary transforms $\{U_{st}\}_{st}$.

In the Schrödinger picture the manipulator is represented by the trail of infinitesimal redefiners $\{R_t\}_t$.

Two kinds of influence

The influence of the other items may be exerted in two different ways.

1. The influence is a result of the mere existence and some attributes of the other item. It depends on the distance between the actor and the subject. Here we take the distance between the locator vectors that characterize these items. The locations of the items are represented in the **coordinate space**. In first instance, we use the coordinate space in order to express the movement of items. This view fits the Schrödinger picture.
2. The influence is a result of observations done by the actor on the position and other characteristics of the subject. The influence depends on the distance between the positions of the items as experienced by the actor. In this case the location of the items are given by the positions of the items in **observed space**. This view fits the Heisenberg picture.

In both cases the influence depends on a distance r . In the two cases these distances have a different origin and relate to different spaces. We will concentrate on the first case.

Redefiner action

One important step must still be taken. In physics observed spacetime has a Minkowski signature. Further we observe that space corresponds with the imaginary part of a position quaternion for which the real part seems to have no physical meaning. We must find an explanation for these facts. The Minkowski signature defines the following time-like relation between the proper time Δt , the space step Δq and the coordinate time step $\Delta \tau$

$$\Delta t^2 = \Delta \tau^2 - \Delta q^2 / c^2 \quad (25)$$

A possible explanation can be given by the action of the redefiner when the action step is perpendicular to the space step and the coordinate time step is used to close the rectangular triangle. The action step Δs equals $c \cdot \Delta t$.

$$\Delta \tau = \Delta s + \Delta q / c \quad (26)$$

This can be achieved when an infinitesimal number transform converts the value of the position observable q as is shown below.

During an infinitesimal step the redefiner moves the subspace such that the current coordinate q_1 is changed to the next coordinate q_2 such that the move corresponds to a number transform:

$$q_2 = u \cdot q_1 \cdot u ;$$

where u is close to unity

$$u \approx 1 + \Delta s.$$

Δs is the infinitesimal action step. We expect the action step Δs to be imaginary. Let us investigate a trail of action steps.

Path characteristics in \mathbb{R}^n

Let $\{\alpha_t\}_t$ describe the path of infinitesimal action steps through a landscape of imaginary quaternions. $\{\alpha_t\}_t$ is a unit speed curve. $\|\alpha'_t\| = 1$ for all t . As an example we will first derive the 3D Frenet-Serret frame.

$$\mathbf{T}_t := \alpha'_t \tag{27}$$

$$\kappa_t := \|\mathbf{T}'_t\| \tag{28}$$

$$\kappa_t \cdot \mathbf{N}_t := \mathbf{T}'_t \tag{29}$$

$$\mathbf{B}_t := \mathbf{T}_t \times \mathbf{N}_t \tag{30}$$

$$\|\mathbf{T}_t\| = \|\mathbf{N}_t\| = \|\mathbf{B}_t\| = 1 \tag{31}$$

\mathbf{T}_t is the tantrix of curve α_t at instance t .

\mathbf{N}_t is the principal normal of curve α_t at instance t . It is only defined when $\kappa_t \neq 0$.

\mathbf{B}_t is the binormal of curve α_t at instance t .

\mathbf{T}_t , \mathbf{N}_t and \mathbf{B}_t are imaginary super-quaternions.

κ_t is the curvature of curve at α_t at instance t .

$r_t = 1/\kappa_t$ is the radius of curvature at instance t .

τ_t is the torsion of curve α_t at instance t .

$$\begin{bmatrix} \mathbf{T}'_t \\ \mathbf{N}'_t \\ \mathbf{B}'_t \end{bmatrix} = \begin{bmatrix} 0 & \kappa_t & 0 \\ -\kappa_t & 0 & \tau_t \\ 0 & -\tau_t & 0 \end{bmatrix} \begin{bmatrix} \mathbf{T}_t \\ \mathbf{N}_t \\ \mathbf{B}_t \end{bmatrix} \tag{32}$$

The Frenet–Serret formulas were generalized to higher dimensional Euclidean spaces by Camille Jordan in 1874. The **Frenet–Serret formulas** for \mathbb{R}^n , stated in matrix language, are

$$\begin{bmatrix} \mathbf{e}'_{t_0} \\ \mathbf{e}'_{t_1} \\ \vdots \\ \mathbf{e}'_{t_{n-1}} \end{bmatrix} = \begin{bmatrix} 0 & \chi_{t_1} & 0 & 0 \\ -\chi_{t_1} & 0 & \chi_{t_2} & 0 \\ 0 & \cdots & & 0 \\ \vdots & \ddots & & \vdots \\ 0 & & 0 & \chi_{t_{n-1}} \\ 0 & \cdots & -\chi_{t_{n-1}} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{e}_{t_0} \\ \mathbf{e}_{t_1} \\ \vdots \\ \mathbf{e}_{t_{n-1}} \end{bmatrix}$$

Here the vectors \mathbf{e}_{t_i} are considered as imaginary vectors in the imaginary subspace \mathbb{R}^n of an 2^m -on space, where $n+1 = 2^m$.

Relativity

Einstein's own explanation of the origin of relativity: "There is no logical way to the discovery of these elementary laws. There is only the way of intuition."

Read more: <http://www.time.com/time/magazine/article/0,9171,878733,00.html#ixzz15NlhpWDu>

The trail that is taken by the actions of the manipulators can be studied with the toolkit of differential geometry. The study of inertia already showed that trails traveled in a geodesic with uniform speed are not affected by inertia. Therefore we take the steps such that the action curve features unit speed and becomes a geodesic.

The tantrix \mathbf{e}_0 of that curve delivers the main part of the action \mathbf{S} . The eigenvalue u can be specified as a function of the trail progression parameter t . The Taylor expansion gives:

$$u_{t+\Delta t} = u_t \cdot (1 + \Delta \mathbf{S}_{rt}) = (1 + \Delta \mathbf{S}_{lt}) \cdot u_t \quad (34)$$

$$\Delta \mathbf{S}_{lt} = \mathbf{e}_{t_0} \cdot \Delta t + \mathbf{e}_{t_1} \cdot \Delta t^2 \cdot \frac{\chi_{t_1}}{2} + \mathbf{e}_{t_2} \cdot \Delta t^3 \cdot \frac{\chi_{t_1} \cdot \chi_{t_2}}{6} \quad (35)$$

$$- \mathbf{e}_{t_3} \cdot \Delta t^4 \cdot \frac{\chi_{t_1} \cdot \chi_{t_2} \cdot \chi_{t_3}}{24} + \mathcal{O}(\Delta t^4) \quad (36)$$

$$u_t = \prod_{\tau=0}^t (1 + \Delta \mathbf{S}_{l\tau})$$

All trail elements u_t are close to unity. The expansion also holds when u_t is a higher dimensional 2^n -on. Since 2^n -ons for $n>1$ don't commute, the ordering of the products is important. $\Delta \mathbf{S}_{rt}$ and $\Delta \mathbf{S}_{lt}$ differ. The tantrix \mathbf{e}_{t_0} is in general not imaginary. The result of the action step is:

$$q_t = u_t^* \cdot q \cdot u_t \approx (1 - \Delta \mathbf{S}_{rt}) \cdot u_{t_0} \cdot q_{t_0} \cdot u_{t_0} \cdot (1 + \Delta \mathbf{S}_{rt}) \quad (37)$$

The real part of q is not affected. So we only consider the imaginary part.

$$\mathbf{q}_t \approx (1 - \Delta s_{rt}) \cdot \mathbf{q}_{t0} \cdot (1 + \Delta s_{rt}) \quad (38)$$

$$\Delta \mathbf{q}_t \approx \mathbf{q}_{t0} \cdot \Delta s_{rt} - \Delta s_{rt} \cdot \mathbf{q}_{t0} - \Delta s_t \cdot \mathbf{q}_{t0} \cdot \Delta s_{rt} \approx 2 \cdot \mathbf{q}_{t0} \times \Delta \mathbf{s}_{rt} \quad (39)$$

Thus the **space step** $\Delta \mathbf{q}_t$ is perpendicular to both \mathbf{q}_{t0} and $\Delta \mathbf{s}_{rt}$. Let us define the local **spacetime step** $\Delta \mathbf{\sigma}_t$ as the action step Δs_{rt} .

$$\Delta \mathbf{\sigma}_t := \Delta \mathbf{s}_{rt} \approx \mathbf{e}_{0rt} \cdot \Delta t \quad (40)$$

The dependence of $\Delta \mathbf{q}_t$ on the size of the inter-distance \mathbf{q}_{t0} is compensated by the dependence of $\Delta \mathbf{s}_{rt}$ on the inverse of that inter-distance. Due to the transformation the **space** step is directed perpendicular to the spacetime step. It is also perpendicular to the stationary part of q . The rectangular triangle formed by $\Delta \mathbf{q}_t$ and $\Delta \mathbf{s}_{rt}$ can be closed with a step that we will relate to the **coordinate time** interval $\Delta \mathbf{\tau}_t$.

$$\Delta \mathbf{\tau}_t := \Delta \mathbf{q}_t / c + \mathbf{e}_{ort} \cdot \Delta t \quad (41)$$

In this setting the factor $2 \cdot \mathbf{q}_{t0}$ is ignored. \mathbf{e}_{ort} is a unit size vector. The formula that relates the three steps corresponds to a time-like convention:

$$|\Delta t|^2 = |\Delta \tau|^2 - |\Delta q|^2 / c^2 \quad (42)$$

The steps define a **Minkowski** metric and as a consequence it raises **special relativity**. The curvature of the manipulator trail causes an additional curvature of the trail of the observed position. The corresponding manifold is not Riemannian, but pseudo-Riemannian. More precisely, it is Lorentzian. The action in the argument of the current manipulator can be related to the gravitation field. The curvature in action space brings general relativity into the picture.

Coordinate time has only local significance. It only exists in combination with an observation of position in Q space. The combination of coordinate time and Q space no longer corresponds to quaternions. These specimens are better treated with Clifford algebras. In this way the number transform introduces other algebras than 2^n -ons.

Speed along the live path

For the speed v_{trail} along the action trail measured in coordinate time units holds:

$$\frac{ds}{d\tau} = \frac{dt}{d\tau} = v_{trail} = \sqrt{1 - (v_{Qspace}/c)^2} \quad (43)$$

Where

$$v_{Qspace} := dq/d\tau \quad (44)$$

is the speed measured in coordinate time units in the Q space along the observed life path of the item.

The action along the live path

The integrated action S_{ab} is performed over a distance along the action trail or equivalently over a period of coordination time

$$\begin{aligned} S_{ab} &= - \int_a^b m \cdot c^2 \cdot ds + \text{matter terms} \\ &= - \int_{\tau_a}^{\tau_b} m \cdot c^2 \cdot \sqrt{1 - \left(\frac{v}{c}\right)^2} \cdot d\tau + \text{matter terms} \\ &= \int_{\tau_a}^{\tau_b} \mathcal{L} \cdot d\tau \end{aligned} \tag{45}$$

m is the mass of the considered item.

v is the speed in Q space.

\mathcal{L} is the Lagrangian.

The first line of this formula can be considered as an integral along the trail in coordinate space or equivalently over the trail in Hilbert space. The next lines concern integrals over the corresponding path in observed space combined with coordinate time. It must be noticed that these spaces have different signature.

$$\mathcal{L} = - m \cdot c^2 \cdot \frac{ds}{d\tau} + \text{matter terms} \tag{46}$$

In general relativity, the first term generalizes (includes) both the classical kinetic energy and interaction with the Newtonian gravitational potential. It becomes:

$$m \cdot c^2 \cdot \frac{ds}{d\tau} = -m \cdot c \cdot \sqrt{g_{\alpha\beta} \cdot \dot{q}_\alpha \cdot \dot{q}_\beta} \tag{47}$$

$g_{\alpha\beta}$ is the rank 2 symmetric metric tensor which is also the gravitational potential. Notice that a factor of c has been absorbed into the square root.

The matter terms in the Lagrangian \mathcal{L} differ from those in the integrated action S_{ab} .

$$S_{ab_matter} = - \int_a^b e \cdot A_\gamma \cdot dq^\gamma + \text{other matter terms} \tag{48}$$

The matter term in the Lagrangian due to the presence of an electromagnetic field is given by:

$$\mathcal{L} = - m \cdot c^2 \cdot \frac{ds}{d\tau} + e \cdot \dot{q}^\gamma \cdot A_\gamma + \text{other matter terms} \tag{49}$$

A_γ is the electromagnetic 4-vector potential.

Fields

It is clear that the physical fields play an important role in nature. They form an indispensable ingredient in the establishment of dynamics.

More fields

As already said, this gravitation field is not the only influence that is supported in the action of manipulators. As shown above, the electromagnetic field is another example. It has the same range than the gravitation field but due to the existence of positive and negative charges the electromagnetic fields of individual items have a tendency to compensate each other. There exists a list of fields with shorter ranges. These are not treated here. If this story is correct, then all these fields have a storage place in the eigenvalues of the manipulators. The action represented by a complete Lagrangian indicates how fields appear in the argument of a manipulator. See [Lagrangian of the world](#) for a complete survey of terms. [Mendel Sachs](#) has found a way to bring all terms under the same hood.

Optics

The optical Fourier transform (OTF) is an objective imaging quality characteristic for imaging devices in a similar way as the frequency transfer function qualifies the signal transfer function of a linearly operating electronic device. The transfer quality of a chain of linear signal transforming devices is characterized by the product of the frequency transfer functions of the elements of the chain. In a similar way the OTF of a chain of imaging devices is given by the product of the OTF's of the elements of the chain. However, this is a profound simplification of reality. The product rule only holds when the transfer characteristics of the imaging devices are spatially uniform over the complete input field of the separate imaging components. Further, the conditions in which the OTF's of the components are determined must be similar to the conditions in the chain. More in detail, this means that the angular distribution, the chromatic distribution and the homogeneity of the radiation must be identical.

In optics, the image sided spread function equals the convolution of the object sided spread function and the point spread function (PSF, the image of a point). The Fourier transform of the image sided spread function is equal to the product of the Fourier transform of the object sided spread function and the optical Fourier transforms (OTF's) of the imaging devices. When several imaging devices work in sequence, then the total optical transfer function of the imaging system equals the product of the transfer functions of the components.

Wave mechanics has much in common with wave optics. For each compact normal operator the Hilbert subspace that represents a physical item corresponds to a spread in Hilbert space and a corresponding spread in the eigenspaces of that normal operator. The distribution of this spread is represented in a [wave function](#). For example the wave function that has the position as a variable corresponds to the triple consisting of a physical item, its Hilbert subspace representation and the position operator.

After a move of a physical item its position related wave function has much in common with the spread function that characterizes the blur of the image sided pictures in a linear operating imaging system. The physical fields that influence the physical item have an equivalent in the chain of imaging devices that transfers the image.

The product formula for the transfer functions relies on several preconditions. First of all it relies on the spatial uniformity of the transfer. At all places where information is passed, the transfer characteristics must be identical. If that is not the case, the product formula has only validity in a small spatial area. The transfer characteristics may be different for each Fourier component. The final result can be computed in longitudinal direction by multiplication and in lateral direction by linearly adding the contributions of these regions in which the transfer is locally uniform. In the summation the angular and chromatic distribution of the transferred

information play a role. These distributions determine the summation coefficients. The extent of the region in which the considered transfer function is considered valid depends on the accuracy that is required for the result of the computation.

In wave mechanics the wave function, which is taken just before the item moves gets the role of the object. After a movement through a region of the fields the wave function has been changed. Its Fourier transform then equals the product of the Fourier transform of the original wave function and the wave transfer functions (WTF's) of the fields that influence the item. If several steps are taken in sequence, then the transfer functions of the passed field pieces must be multiplied in order to get the overall result. This transfer is affected in a similar way by spatial non-uniformity as the optical case.

In cylindrical imaging systems Seidel aberrations take their toll. When the system is folded or when lenses are not perfectly in line, also non-cylindrical influences will influence the imaging quality. The measurement and the specification of the OTF must cope with the spatial non-uniformity of the imaging characteristics of the imaging devices and with the angular and chromatic distribution of the radiation. The OTF also depends on the longitudinal location of the object and where the image is detected. This also occurs with the WTF of physical fields. Both in optics and in wave mechanics the precise locations of the "object" and the "image" are not well determined. They are defined by spatial distributions. In both cases the angular and chromatic distributions of the contributing radiation influence the transfer. The final result is constituted by the weighted sum of all contributions.

From optics it is known that the modulation transfer function (MTF) is a proper imaging qualifier for inhomogeneous light imaging. In inhomogeneous imaging the imaging process can be properly described by ray tracing. Ray tracing has much similarities with the application of the path integral. However, ray tracing normally does not use arbitrary paths. In inhomogeneous imaging phases are scrambled. For holographic imaging the phase transfer function (PTF) or the whole OTF is the better measure. With holographic imaging the phases carry the depth information. Feynman's path integral can cover arbitrary paths because interference via the phases eliminates the contributions of non-realistic paths. That is why in the path integral the angular distribution of the radiation plays no role.

In optics the OTF depends on the position in the object space. Off axis the OTF is not rotationally symmetric. The OTF also depends on the angular distribution and the chromatic distribution of the radiation. These dependencies also hold for the WTF in wave mechanics. They become noticeable when a longer path is travelled.

A longitudinal displacement of the image spread function with respect to the object spread function corresponds to an extra phase term in the longitudinal component of the Fourier transform of the image spread function. A lateral displacement corresponds to an extra phase term in the transverse component of the Fourier transform. In wave mechanics this holds for the respective components of the Fourier transform of the wave function after the move.

The resemblance between optics and wave mechanics becomes striking when the discrete lens pack is replaced by a medium with a continuously varying refraction.

Systems

A system is a local assembly of physical items that act as a single physical item. Its [state](#) is mixed. When a redefinition of physical items in terms of atomic propositions goes together

with influences between items in the form of fields, then a redefinition of a system in terms of its components will certainly also have such effects. The redefinition may take different forms. It may be represented by an emission or absorption of a component or it may be a reshuffling of the components. The simplest case of reshuffling is a permutation of items that belong to the same category. A more complex situation is a periodic movement of one or more components within the realm of a system. In addition each sequence of creation and annihilation is a form of redefinition.

The system has its own characteristic vectors. The wave function may depend on the permutation state of the system. For example for fermions an odd permutation changes the sign of the (position related) wave function. For bosons a permutation does not affect the wave function. Permutations of different categories of components go together with their own type of influence. Thus, there are fermionic fields and there are bosonic fields. Each of these fields has its own type of creation and The annihilation. Being fermion or boson relates to the spin type of the component. The annihilation and creation operators are closely related to the type of components involved and are also closely related to the type of fields involved. The annihilation/creation operators of fermions anti-commute and the annihilation/creation operators of bosons commute.

Entropy

A [system](#) is a local assembly of [physical items](#) that act as a single physical item. The Density operator ρ relates to the currently considered observable Q . A pure state is a ray spanned by an eigenvector of the operator Q .

[The von Neumann entropy](#) $S(\rho)$ of a physical system that is characterized by a [state](#) $|\psi\rangle$ is given by

$$\rho = \sum_q \{|q\rangle \langle q|\} = \sum_q \{\lambda_q \cdot \rho_q\}$$

$$\rho_q = |q\rangle \langle q|$$

$$\lambda_q = |\langle \psi | q \rangle|^2$$

$$S(\rho) = -k_B \cdot \sum_q \{\lambda_q \cdot \ln(\lambda_q)\}$$

The entropy $S(\rho)$ describes the departure of the system from a pure state. In other words, it measures the degree of mixture of the state $|\psi\rangle$.

Some properties of the von Neumann entropy:

- $S(\rho)$ is only zero for pure states.
- $S(\rho)$ is maximal and equal to $\log_2 N$ for a maximally mixed state, N being the dimension of the Hilbert space.
- $S(\rho)$ is invariant under changes in the basis of ρ , that is, $S(\rho) = S(U\rho U^\dagger)$, with U a unitary transformation.
- $S(\rho)$ is concave, that is, given a collection of positive numbers λ_q which sum to unity ($\sum_q \lambda_q = 1$) and density operators ρ_q , we have

$$S\left(\sum_q \lambda_q \rho_q\right) \geq \sum_q \lambda_q S(\rho_q)$$

- $S(\rho)$ is additive. Given two density matrices ρ_A, ρ_B describing independent systems A and B , then

$$S(\rho_A \otimes \rho_B) = S(\rho_A) + S(\rho_B)$$

Instead, if ρ_A, ρ_B are the reduced density operators of the general state ρ_{AB} , then

$$|S(\rho_A) - S(\rho_B)| \leq S(\rho_{AB}) \leq S(\rho_A) + S(\rho_B)$$

While in Shannon's theory the entropy of a composite system can never be lower than the entropy of any of its parts, in quantum theory this is not the case, i.e., it is possible that $S(\rho_{AB}) = 0$ while $S(\rho_A) > 0$ and $S(\rho_B) > 0$.

Intuitively, this can be understood as follows: In quantum mechanics, the entropy of the joint system can be less than the sum of the entropy of its components because the components may be [entangled](#). The left-hand inequality can be roughly interpreted as saying that entropy can only be canceled by an equal amount of entropy. If system A and system B have different amounts of entropy, the lesser can only partially cancel the greater, and some entropy must be left over. Likewise, the right-hand inequality can be interpreted as saying that the entropy of a composite system is maximized when its components are uncorrelated, in which case the total entropy is just a sum of the sub-entropies.

- The von Neumann entropy is also *strongly sub-additive*. Given three Hilbert spaces, A, B, C ,

$$S(\rho_{ABC}) + S(\rho_B) \leq S(\rho_{AB}) + S(\rho_{BC})$$

Isolated systems

With isolated systems we mean systems in a geometrically compound environment where influences from the environment compensate each other, possibly including the influences on the environment that are caused by the system under consideration. This includes e.g. the gravitation field. Internal influences are internally compensated such that they are not felt by other systems. For example the sum of the charges, which are related to electromagnetic fields is zero. It means that the Fourier transforms of the local fields consist of linear combinations of discrete terms. This holds for the electrostatic fields and the magneto-static fields. It holds for rectangular components as well as for polar components. These components are the germs of quanta and are the source of creations and annihilations.

For example consider the vector potential A . Its Fourier transform can be written as:

$$A(\mathbf{r}, t) =$$

$$\sum_k \sum_{\mu=-1,1} \{ \mathbf{e}_{\mu(k)} \cdot a_{\mu k}(t) \cdot \exp(i(\mathbf{k}, \mathbf{r})) + \bar{\mathbf{e}}_{\mu(k)} \cdot \bar{a}_{\mu k}(t) \cdot \exp(-i(\mathbf{k}, \mathbf{r})) \}$$

Where \mathbf{e}_μ are unit sized polarization vectors. They depend on the orthonormal vectors \mathbf{e}_x and \mathbf{e}_y that represent quaternionic imaginary base numbers. The index μ labels the photon spin. The product $\mathbf{e}_\mu \cdot \mathbf{a}_\mu$ represents a quaternionic imaginary number. The number i can be interpreted as a base imaginary number in the direction of \mathbf{k} .

$$\mathbf{e}_1 \equiv \frac{-1}{\sqrt{2}}(\mathbf{e}_x + i \cdot \mathbf{e}_y)$$

$$\mathbf{e}_{-1} \equiv \frac{1}{\sqrt{2}}(\mathbf{e}_x - i \cdot \mathbf{e}_y)$$

$$(\mathbf{e}_x, \mathbf{k}) = 0$$

$$(\mathbf{e}_y, \mathbf{k}) = 0$$

$$[a_\mu(\mathbf{k}), a_{\mu'}(\mathbf{k}')] = 0$$

$$[a_\mu^\dagger(\mathbf{k}), a_{\mu'}^\dagger(\mathbf{k}')] = 0$$

$$[a_\mu(\mathbf{k}), a_{\mu'}^\dagger(\mathbf{k}')] = \delta_{\mu\mu'} \cdot \delta_{\mathbf{k}\mathbf{k}'}$$

Here the $\sqrt{\frac{\hbar}{2\omega V \epsilon_0}} a_\mu(\mathbf{k})$ are the operator equivalents of the coefficients $a_{\mu\mathbf{k}}$ and $\omega = c |\mathbf{k}| = ck$.

This results in:

$$\mathbf{A}(\mathbf{r}, t) = \sum_{\mathbf{k}, \mu} \sqrt{\frac{\hbar}{2\omega V \epsilon_0}} \{ \mathbf{e}_{\mu(\mathbf{k})} \cdot a_\mu(\mathbf{k}, t) \cdot \exp(i(\mathbf{k}, \mathbf{r})) + \bar{\mathbf{e}}_{\mu(\mathbf{k})} \cdot a_\mu^\dagger(\mathbf{k}, t) \cdot \exp(-i(\mathbf{k}, \mathbf{r})) \}$$

$$\mathbf{E}(\mathbf{r}, t) = i \cdot \sum_{\mathbf{k}, \mu} \sqrt{\frac{\hbar}{2\omega V \epsilon_0}} \{ \mathbf{e}_{\mu(\mathbf{k})} \cdot a_\mu(\mathbf{k}, t) \cdot \exp(i(\mathbf{k}, \mathbf{r})) - \bar{\mathbf{e}}_{\mu(\mathbf{k})} \cdot a_\mu^\dagger(\mathbf{k}, t) \cdot \exp(-i(\mathbf{k}, \mathbf{r})) \}$$

$a_\mu(\mathbf{k}, t)$ is an annihilation operator and $a_\mu^\dagger(\mathbf{k}, t)$ is a creation operator.

$$a_\mu^\dagger(\mathbf{k}, t) |n\rangle = |n+1\rangle \sqrt{n+1}$$

$$a_\mu^\dagger(\mathbf{k}, t) |0\rangle = |1\rangle$$

$$a_\mu(\mathbf{k}, t) |n\rangle = |n-1\rangle \sqrt{n}$$

$$a_\mu(\mathbf{k}, t) |0\rangle = 0$$

$$[a_\mu(\mathbf{k}), (a_\mu^\dagger(\mathbf{k}))^n] = (a_\mu(\mathbf{k}))^n$$

The Hamiltonian is:

$$H(t) = \hbar\omega \sum_{\mathbf{k}, \mu} \{a_{\mu}^{\dagger}(\mathbf{k}, t) \cdot a_{\mu}(\mathbf{k}, t) + 1/2\}$$

The number operator N_{μ} gives the number of quanta:

$$N_{\mu}(\mathbf{k}, t) = a_{\mu}^{\dagger}(\mathbf{k}, t) \cdot a_{\mu}(\mathbf{k}, t)$$

The quanta discussed here are bosons. With the electromagnetic field they are photons. Photons have integer spin 1. With the dyadic product \otimes follows:

$$S_z \equiv -i\hbar(\mathbf{e}_x \otimes \mathbf{e}_y - \mathbf{e}_y \otimes \mathbf{e}_x) \text{ and cyclically for } x \rightarrow y \rightarrow z \rightarrow x$$

$$[S_x, S_y] = i\hbar S_z$$

$$S_z \cdot \mathbf{e}_{\mu} = \mu \cdot \mathbf{e}_{\mu}$$

Fermions have half integer spin. With fermions the creation and annihilation operators a and a^{\dagger} have different commutation relations. Instead of commuting, these operators anti-commute.

Spin and dyadic product

As factors of the dyadic product we consider imaginary quaternionic numbers or vectors in \mathbb{R}_3 . The product corresponds to a matrix. This matrix acts as an operator.

$$u \otimes v \rightarrow \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} = \begin{bmatrix} u_1 v_1 & u_1 v_2 & u_1 v_3 \\ u_2 v_1 & u_2 v_2 & u_2 v_3 \\ u_3 v_1 & u_3 v_2 & u_3 v_3 \end{bmatrix}$$

The product of quaternions contains sign selections. For the imaginary parts this selection has to do with the handedness of the external product. Dyadic products are well suited to store the product such that the sign selections are stored as well. The sign selection plays its role in the dyad \mathbf{ij} , which consists of two imaginary base numbers. The dyad $\mathbf{ij} = -\mathbf{ji}$, and \mathbf{k} can be $\pm \mathbf{ij}$. Let us apply this to the definition of S_z .

$$S_z = -i\hbar \begin{bmatrix} 0 & \mathbf{e}_x \mathbf{e}_y - \mathbf{e}_y \mathbf{e}_x & 0 \\ \mathbf{e}_y \mathbf{e}_x - \mathbf{e}_x \mathbf{e}_y & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = i\hbar \begin{bmatrix} 0 & -2\mathbf{e}_x \mathbf{e}_y & 0 \\ 2\mathbf{e}_x \mathbf{e}_y & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

This shows that the definition of S_z via the dyadic product reflects the choice in handedness of the external product of \mathbf{e}_x and \mathbf{e}_y .

The proposition

This finding indicates that when our interpretation of Sciama's analysis is correct, the original proposition

All items in universe influence each other's position.

is not generally true. The universe of items does not influence position. It counteracts acceleration of individual items. Position is only influenced in an indirect way and presupposes an observation. If the item moves in a geodesic with uniform speed, then the position changes while the influences of all other items compensate each other. In such cases the summed influence is zero.

We may alter the original proposition (*). If our analysis is correct, then the proposition
All items in universe influence each other's acceleration.
is true. Here acceleration is measured in the coordinate space.

Time

We have encountered several notions of time. The first is the trail progression parameter. We named it manipulator time. The manipulator action together with the observation of position cause a quaternion waltz. The geometry of this waltz invites to define both a spacetime step and a coordinate time step. The manipulator time has more resemblance with proper time. The actions of the manipulator are not confronted with a maximum speed. However, observations that apply the suggested coordinate time are definitely confronted with the maximum speed c of information transfer.

Dynamic observables

Like the manipulator an observable can be interpreted as a dynamic operator. As such it can be mimicked by a trail of static observables. Where the dynamic manipulator can be given a global scope, the dynamic observable will only have a local scope. It depends on who is observing and what is observed. Via the number waltz it also depends on the local characteristics of the manipulator. The observer can be another item or the representative of a set of items. Observed are eigenvalues of manipulated normal operators. After manipulation the trail of observed values get curved. This curvature is directly related to the curvature of the actions of the manipulator. The manipulator gives the observed values an extra “turn”.

Measurement preparation

In a measurement the observation follows after a preparation phase by the measuring equipment. Such a preparation may change the shape of the subspace that represents the item. For example, a preparation for precise position measurement may squeeze the item's subspace such that its range of covered momentum eigenvectors extends very far. Similarly, when a preparation is made for precise momentum measurement then the item's subspace is squeezed in the other direction, such that it covers a huge range of position eigenvectors. A Fourier transform also squeezes the item's subspace.

Related stuff

Quaternionic argument

The logarithmic of quaternions is not well defined. However, this function is well defined in the complex subspaces of a quaternionic space. This has a direct effect on those constructs that make use of this concept. Examples are canonical conjugates and Fourier transforms.

The coordinate space and the action space use the combination of a directional unit size imaginary number and a real argument value in order to define their elements. Both spaces are imaginary spaces. The elements in coordinate space are images of points that are uniformly distributed on the unit sphere of the quaternions. The relevant elements in action space, those that represent the characteristic values of items, form a curved manifold. These arguments are arguments of $2n$ -ons. Locally they resemble arguments of unit size quaternions. On even smaller locations, thus in a complex subspace the argument follows from the logarithm.

Momentum operator

The momentum P is the canonical conjugate of the position operator Q . It is also a generator of displacement. It can be defined in a complex subspace of a quaternionic number field that is used to specify inner products. It is defined by specifying the function that defines the inner products of the eigenvectors $|q\rangle$ of Q and $|p\rangle$ of P with real eigenvalues q and p .

$$\langle q|p\rangle = \langle p|q\rangle^* = \exp(\hat{\mathbf{i}} \cdot p \cdot q / \hbar) \quad (\text{R1})$$

The constant \hbar is Planck's constant and relates to the granularity of the eigenspaces. The imaginary base number $\hat{\mathbf{i}}$ belongs to the complex subspace. This specification also defines a complex Fourier transform. The Fourier transform U_{qp} converts the base $\{|q\rangle\}_q$ into the base $\{|p\rangle\}_p$. The inverse Fourier transform U_{pq} does the reverse.

$$\langle q|f\rangle = \sum_p (\langle q|p\rangle \cdot \langle p|f\rangle) \quad (\text{R2})$$

$$= \sum_p \langle p (\langle p|q\rangle) |f\rangle$$

$$= \sum_p \langle p U_{pq} |f\rangle$$

$$= \sum_p \langle p |U_{qp} f\rangle$$

$$\langle p|f\rangle = \sum_q (\langle p|q\rangle \cdot \langle q|f\rangle) \quad (\text{R3})$$

$$= \sum_q \langle q |U_{pq} f\rangle$$

Due to its specification, the momentum operator P can be interpreted as a displacement generator.

$$P = \hat{\mathbf{i}} \cdot \hbar \cdot \frac{\partial}{\partial q} \quad (\text{R4})$$

The Fourier transform is the source of the existence of the Heisenberg uncertainty relation. This is also shown in the commutator:

$$[P, Q] = PQ - QP = \hat{\mathbf{i}} \cdot \hbar \quad (\text{R5})$$

If Q is the position observable, then:

- The observable Q and its base of eigenvectors $\{|q\rangle\}_q$ represents the **particle view** of the item.
- The observable P and its base of eigenvectors $\{|p\rangle\}_p$ represents the **wave view** of the item.

In the restricted view of the complex subspace the operators Q and P are taken as Hermitian.

In the quaternionic space their equivalents \mathbf{Q} and \mathbf{P} are anti-Hermitian operators and will usually have different imaginary directions, which also differ from $\hat{\mathbf{I}}$. It is save to conclude that the imaginary direction $\hat{\mathbf{I}}$ is related with the Fourier transform U_{pq} rather than with Q or P .

The operator $\hat{\mathbf{I}}_t$ makes the complex Fourier transform U_{pq} into a dynamic quaternionic Fourier transform U_{pqt} . In this way, it gets a dynamic imaginary direction. In the same sense the operators \mathbf{Q}_t and \mathbf{P}_t can be considered as quaternionic operators. In that case they must also be given a direction. \mathbf{P}_t can be given a direction that is constituted from a uniform movement \mathbf{v} in the direction parallel to the action step $\Delta\mathbf{g}_t$ and the current displacement $\Delta\mathbf{q}_t$ that is due to the waltz. $\Delta\mathbf{q}_t$ is perpendicular to $\Delta\mathbf{g}_t$ and \mathbf{Q}_t .

We can state:

$$\begin{aligned} [\mathbf{P}_t, \mathbf{Q}_t] &= \mathbf{P}_t \cdot \mathbf{Q}_t - \mathbf{Q}_t \cdot \mathbf{P}_t \\ &= U_t^\dagger \cdot (\mathbf{P} \cdot \mathbf{Q} - \mathbf{Q} \cdot \mathbf{P}) \cdot U_t = U_t^\dagger \cdot \hat{\mathbf{I}} \cdot U_t \cdot \hbar = \hat{\mathbf{I}}_t \cdot \hbar \end{aligned} \quad (\text{R6})$$

\mathbf{P}_t not only contains the result of the waltz. It also contains any uniform motion that runs unaffected from the action of U_t . That part of \mathbf{P}_t is not affected by inertia.

\mathbf{P} and \mathbf{Q} possess a fixed relation through the definition of the canonical conjugate. The Fourier transform can be interpreted as the local appearance of a more complicated 2^n -on operator. This means that the canonical conjugate must be redefined at every location of the trail.

Heisenberg's uncertainty

The definition of the canonical conjugate incorporates the Heisenberg's uncertainty principle. It means that a small environment of q values goes together with a large spread of p values. And visa versa.

$$\Delta q \cdot \Delta p \geq \hbar/2 \quad (\text{R7})$$

This has direct consequences for the (re)definition of the subspace that represents an item. Here Heisenberg's uncertainty principle translates in a corresponding relation between eigenvectors of \mathbf{Q} and \mathbf{P} . This relation can best be stated in coordinate space. The item must cover a region that corresponds to the above equation.

Entanglement

Two or more items may overlap. It means that the subspaces that represent these items overlap. Two items can overlap even when their positions are quite distant from each other. If those positions are rather sharply defined, then their momentum will spread such that the items still may overlap. This is the background of the concept of **entanglement**.

Wave mechanics

The wave view reveals that there exist a strong relation between optics and quantum physics. In many aspects wave mechanics resembles Fourier optics, but then taken one dimension higher. In this respect the hole in the camera obscura has a direct relation with the holographic screen that surrounds a black hole.

Lenses act like Fourier transformers. Their principal planes can be considered as holographic screens. A holographic screen is a position where the amplitudes of the waves are zero. At this place all information is transferred via phases. At the object and image points the information transfer runs via amplitudes.

The Optical Transfer Function (OTF) is used to qualify the Fourier transform quality of imaging devices. Its modulus the Modulation Transfer Function (MTF) is used to qualify imaging with inhomogeneous light. That is light in which the phases are scrambled. This kind of imaging can be well described with light rays, thus imaging with light particles (photons). Information transfer is done via amplitudes.

Holographic imaging is best described with the full OTF. That function includes both the MTF and the Phase Transfer Function (PTF). The PTF is best suited to qualify the behavior with interference, thus with waves. This indicates that with holographic information transfer the phases play an important role.

With inhomogeneous imaging the phases are ignored. These facts indicate the difference between the particle view and the wave view.

Movement of the item

Here we do not mean the observed movement, but the unobserved moves caused by the trail of manipulators. These movements follow a geodesic on the manifold of eigenvalues of the manipulators. This movement is characterized by a [geodesic equation](#). However this equation runs in coordinate space rather than observed space:

$$\frac{d^2 x^a}{dt^2} + \Gamma_{bc}^a \cdot \frac{dx^b}{dt} \cdot \frac{dx^c}{dt} = 0 \tag{R8}$$

where Γ_{bc}^a are the Christoffel symbols of the metric. The parameter t is the manipulator time. The Christoffel symbols are functions of the metric and are given by:

$$\Gamma_{bc}^a = \frac{1}{2} \cdot g^{ad} \cdot (g_{cd,b} + g_{bd,c} - g_{bc,d}) \tag{R9}$$

where the comma indicates a partial derivative with respect to the coordinates:

$$g_{ab,c} = \frac{\partial g_{ab}}{\partial x^c} \tag{R10}$$

The section on [relativity](#) shows how the movement of the item transfers to the movement of observed quantities.

Measurement

We differentiate between a measurement using a piece of equipment and an observation as is done between items in universe. In the particle view the measuring equipment scrambles the

phases. After that scrambling an observation is done. In the wave view the measuring equipment takes care that the phases stay intact, while the amplitudes are ignored during the next observation.

In measurement terms the scramble of the phases is called **de-coherence**. In the same sense the care to keep phases pure and the neglecting of the amplitudes could be called re-coherence. Both actions can be related with the Fourier transforms that convert the wave view into the particle view or visa versa.

Spacetime and the Hamiltonian

According to our analysis, space and time are not closely related. The relation via spacetime is a rather artificial construct. It involves the outcome of the quaternion waltz! However, physicists tend to see this relation as fundamental. Our analysis shows that the Hermitian part of the position operator does not appear in the dynamics of the model. Instead the trail progression parameter appears to be the source of proper time. For the Hermitian part of the quaternionic canonical conjugate P of the position operator (the momentum operator) holds the same as for the Hermitian part of the quaternionic position operator Q . It does not appear in the model! Physicists place the Hamiltonian in this non-used place. However, the Hamiltonian has more to do with the manipulator than with the momentum operator. On the other hand, the momentum operator is a displacement generator. In that way it is indirectly related (via the waltz) with the current manipulator.

Hamilton-Jacobi

The Hamilton-Jacobi equation shows how the Hamiltonian relates to the action S of the current manipulator. In this section we consider t to be the manipulator time!

$$H \cdot U_t = \tilde{I}_t \cdot \hbar \cdot \frac{\partial U_t}{\partial t} \quad (\text{R11})$$

For the eigenvalues holds

$$\Delta u_t \approx \Delta S_{lt} \cdot u_t \quad (\text{R12})$$

Thus, we can put

$$H \cdot U_t = - \left(\frac{\partial S_t}{\partial t} \right) \cdot U_t \quad (\text{R13})$$

$$H = - \left(\frac{\partial S_t}{\partial t} \right) \quad (\text{R14})$$

For the expectation values s_t of the action operator S_t holds

$$\tilde{I}_t \cdot \hbar \cdot \frac{\Delta S_t}{\Delta t} = \mathbf{e}_{t0} + \mathbf{e}_{t1} \cdot \Delta t \cdot \frac{\chi_{t1}}{2} + \mathbf{e}_{t2} \cdot \Delta t^2 \cdot \frac{\chi_{t1} \cdot \chi_{t2}}{6} \quad (\text{R15})$$

$$- \mathbf{e}_{t3} \cdot \Delta t^3 \cdot \frac{\chi_{t1} \cdot \chi_{t2} \cdot \chi_{t3}}{24} + \mathcal{O}(\Delta t^3)$$

This derivation is completely independent from the observation of Q. Thus S_t has nothing to do with the Minkowski metric that appears during observations of position.

Hamiltonian as generator

Like P is H a generator. Thus, there is a corresponding canonical conjugate and a corresponding Fourier transform. Proper candidates would be the trail progression parameter t and the current manipulator U_t . However, it is common practice to see H as the generator of the coordinate time τ . The current manipulator may fit as the corresponding Fourier transform. In that case it is a very complicated Fourier transform that only locally acts as our common notion of a Fourier transform. The eigenfunctions of this transform play a very particular role. They form the base of the harmonic movements that occur in the internals of the items. See [S. Thangavelu](#).

For the manipulator time the selection of the sign of the real axis has the meaning of time reversal.

The Lagrangian

The Lagrangian \mathcal{L}_τ is related with the action s_t .

$$s_t = \int_a^b \mathcal{L}_\tau d\tau \quad (\text{R16})$$

The integral is taken over the trail with the observed path. The index t of the action S_t is the trail progression parameter. The integration parameter stands for the coordinate time. The right side of the equation plays in Lorentzian space.

The Euler Lagrange equations explicitly use observations. For that reason the Lagrangian is considered to be a function of the observed q , the velocity \dot{q} and the coordinate time τ . The velocity is measured with the coordinate time.

$$\mathcal{L}_\tau = \mathcal{L}_\tau(\tau, q, \dot{q}) \quad (\text{R17})$$

$$\dot{q} = \frac{dq}{d\tau} \quad (\text{R18})$$

The Euler-Lagrange equations are:

$$\frac{\partial \mathcal{L}_\tau(\tau, q, \dot{q})}{\partial q_i} - \frac{d}{d\tau} \frac{\partial \mathcal{L}_\tau(\tau, q, \dot{q})}{\partial \dot{q}_i} = 0 \quad (\text{R19})$$

for $i = x, y, z$

When the Lagrangian does not vary with one or more of its parameters, then this corresponds with a symmetry of the system. By [Noether's theorem](#), such symmetries of the system correspond to [conservation laws](#). In particular, the invariance of the Lagrangian with respect to time τ implies the conservation of energy.)

By partial differentiation of the above Lagrangian, we find:

$$\frac{\partial \mathcal{L}_\tau(\tau, q, \dot{q})}{\partial q_i} = \frac{\partial U}{\partial q_i} = F_i \quad (\text{R20})$$

$$\frac{\partial \mathcal{L}_\tau(\tau, q, \dot{q})}{\partial \dot{q}_i} = m \cdot \dot{q}_i = p_i \quad (\text{R21})$$

where the force is $\mathbf{F} = -\nabla U$ (the negative gradient of the potential, by definition of conservative force), and \mathbf{p} is the momentum. By substituting these into the Euler–Lagrange equation, we obtain a system of second-order differential equations for the coordinates on the particle's trajectory,

$$F_i = \frac{d(m\dot{q}_i)}{dt} = m \cdot \ddot{q}_i = \dot{p}_i \quad (\text{R22})$$

which is Newton's second law.

The world's action

The action S_t represents the influences that the rest of the world via unitary operator U_t release onto the state $\{|f\rangle_s\}_s$.

In his book about quantum gravity Rovelli writes:

"In the general relativistic parlance 'matter' is anything which is not the gravitational field. As far as current physics knows, the world is made up of the gravitational field, Yang Mills fields, fermion fields and, presumably, scalar fields."

(Carlo Rovelli, book: Quantum gravity, 2004, chapter 2, paragraph 2.1.2)

All these fields give a contribution to the action S .

$$S(e, \omega, A, \psi, \varphi) = S_{\text{GR}}[e, \omega] + S_{\text{matter}}[e, \omega, A, \psi, \varphi] = S_{\text{GR}}[e, \omega] + S_{\text{YM}}[e, A] + S_{\text{f}}(e, \omega, A, \psi) + S_{\text{sc}}[e, A, \varphi] \quad (\text{R23})$$

e is the gravitational field.

$A(q)$ is the electromagnetic field.

$\omega(q)$ is the spin connection. It is a one form in the Lie algebra of the Lorentz group $so(3,1)$

$\psi(q)$ is a scalar field, possibly with values in the representation of the Yang Mills group.

$\varphi(q)$ is a field in the spinor representation of the Lorenz group.

$A(q)$ has a non Abelian connection to the Yang Mills group.

The local characteristics of these fields must be represented in the eigenvalue of the current manipulator.

Representing multiple fields

Professor Mendel Sachs recently wrote a few books in which he promotes the inclusion of more terms in the metric than Einstein did. Sachs uses a four vector with quaternionic coefficients in order to specify the metric. Sachs uses all sixteen terms, while Einstein skipped six due to symmetry considerations. The argument of Sachs is that the symmetry is broken due to the characteristics of the quaternion number field. See:

<http://www.compukol.com/mendel/publications/publications.html>.

16-ons contain the required 16 real numbers that can be arranged as a four vector with quaternion coefficients. Sachs still uses the Minkowski metric. So, his view concerns observed space.

Manipulated observations

When we study the differential geometry of the manipulation, then we stay in a Riemannian environment. In this way we can get equations of motion for the considered item. However, these equations do not concern observed data. The number waltz that represents the manipulation of the observable transfers these equations to our observable world. This two step operation should lead to the equations that characterize (macro) quantum dynamics.

Discussion

Tricks and consequences

We have successfully introduced special relativity into our model.

By introducing relativity the way we did we played a few tricks.

- We used the Schrödinger picture rather than the Heisenberg picture.
 - With respect to the position observable q we assume that a number transform with a number close to unity implements the action of the redefiner step.
- We neglect a significant part of each observable
 - We do not consider the progression parameter as an observable.
 - We do not consider action as an observable.
 - The same holds for proper time
 - We neglect the real part of the position observable. It plays no part in dynamics.
 - We neglect part of the imaginary part of the position observable.
 - We ignore the geometric relation $\Delta \mathbf{q}_t \approx 2 \cdot \mathbf{q}_{t0} \times \Delta \mathbf{s}_{rt}$ (the size of \mathbf{q}_{t0} is compensated)
 - We ignore who or what is observing.
- We shift from progression parameter t to coordinate time τ .
- We shift from Hilbert space via coordinate space to observed space, thereby losing one dimension.
- We combine observed space with coordinate time into a Minkowski/Lorentzian space.
- We relate two integrations into very different spaces. See the remark with formula (45).
- We introduce the mass m in order to indicate the strength of the action.

As a consequence

- We shift from 2^n -on/Riemannian space to Minkowski/Lorentzian space.
- We are confronted with Clifford, Jordan and Grassmann algebras instead of 2^n -on algebras.
- With these algebras we can use complex analysis instead of the more complicated $2n$ -on analysis.
- We are confronted with unintuitive selection features.
- In the new space the waltz becomes an odd operation.
- We need spinors in order to cope with these facts.

Can we do without relativity?

Yes.

- Skip coordinate time.
- Buy a different clock that measures manipulator time.
- Take the waltz serious.
 - Keep the relation $\Delta \mathbf{q}_t \approx 2 \cdot \mathbf{q}_{t0} \times \Delta \mathbf{s}_{rt}$.
- Find out who or what is observing.
 - Where has \mathbf{q} its origin?

However, you would have to fight existing conventions.

A full circle

The usual critique on Sciama's analysis that he neglects the bounded speed of information transfer, do not hold in our setting. We are not using observations to implement influences.

The only critique that remains can be that we use the form of the gravity field $f = -k/r$ in order to derive inertia. We use inertia to explain the occurrence of the field in the action of the current manipulator. Next we claim that the current manipulator generates gravity.

Macro and micro

The treatise up to so far confines to macroscopic dynamics. Micro dynamics concerns movements that occur inside the representation of small particles. Thus, inside the subspace that represent the particle.

In order to stay inside the item, the internal movements must be periodical. They can be combinations of oscillations and rotations. The harmonics oscillator and the spherical harmonics are a well known examples.

The current manipulator can be seen as a very complicated Fourier transform. The eigenfunctions of quantum harmonic movements seem to be governed by the eigenfunctions of this manipulator. Thus micro dynamics occurs via a different process.

Harmonics

For the quantum harmonic oscillator the eigenfunctions and eigenvalues become

$$\langle u | f_n \rangle = c \cdot \exp(-u^2/2) \cdot H_n(u) \quad (R24)$$

where

$$|u\rangle = |x\rangle \cdot \sqrt{(m \cdot \omega) / \hbar} \quad (R25)$$

$$c = 1 / \sqrt{(2^n \cdot n!)} \pi^{-1/4} \quad (R26)$$

$H_n(u)$ are the Hermite polynomials.

The functions $\langle u | f_n \rangle$ are eigenfunctions of the complex Fourier transform.

The total energy is

$$E_n = n + 1/2 \quad (R27)$$

[S. Thangavelu](#) describes eigenfunctions of Fourier transforms in \mathbb{R}^n .

Hilbert space dynamics

Without the unitary transforms or their extensions: the manipulators and without the sign selections the Hilbert space is a static body. Also in this static condition the set of closed subspaces will still be characterized as an orthomodular lattice that is congruent with quantum logic.

The driving forces behind the manipulators are the influences which concretize in physics as fields. These influences are caused by the items as a reaction on accelerations that can be interpreted as unordered replacements of atomic propositions in more complicated propositions that represent items.

Canonical conjugates and the corresponding Fourier transforms cause the particle/wave duality that characterizes quantum behavior. The orthomodular structure of the set of subspaces is required for the existence of this feature. The relation between canonical conjugates and displacement generators connects them to the dynamics of quanta. Remember that the Fourier transform is a particular kind of unitary transform.

Conclusion

Quantum logic is only a partial description of the fundamentals of quantum physics. It only describes the static skeleton in which the quantum dynamics takes place. An important ruler of quantum dynamics is the influence that is exposed by the universe of items in the phenomenon inertia. It indicates the laws that govern the exchange of atomic propositions from enveloping propositions. This has to do with the dynamics of the logic that describes quantum physics. The number waltz is just a property of the number field that is used to define the Hilbert space.

The dynamics of the unobserved life path of an item can be described by a geodesic equation that concerns a Riemannian manifold of 2^n -ons that locally resemble quaternions or in a still smaller region resemble complex numbers. The observed live path differs from the unobserved life path. In fact, each type of continuous observable has its own life path.

For macroscopic dynamics the number waltz appears to play a fundamental role and is the source of special relativity. The waltz applies when unitary transforms affect observations. The source of general relativity is formed by the influences of other items on the considered item. It is expressed in the actions of the current manipulator. As long as no observations are done, special relativity has no impact on these actions.

There are strong indications that in our model there exists a universe wide clock. This is the manipulator time clock. If this is the case, then redefinitions are universe wide synchronized. There are also strong indications that in our model universe is controlled by a single dynamic manipulator. However, its eigenvalues are controlled by the influences that are generated by the local items.

Microscopic movements are governed by a different process. They are directly controlled by the current manipulator and relate to its eigenfunctions.

Trying to implement a complex quantum logical proposition in Hilbert space is indeed an elucidating experience.

Appendix

History of quantum logic

Around 1930 John von Neuman and Garrett Birkhoff were searching for an acceptable explanation of the results of experiments that showed that the execution of an observation of a very small object can completely destroy the validity of an earlier observation of another observable of that object. The Schrödinger equation that agreed with the dynamic behaviour of the particles already existed. Not much later Heisenberg's matrix formulation became popular as well. Quite soon the conclusion was made that something was fundamentally wrong with the logic behind the behaviour of small particles. These small objects show particle behaviour as well as wave behaviour and they show quantization effects. It was found that the distribution axiom of classical logic had to be changed. Soon it became apparent that the lattice structure of classical logic must be weakened from an orthocomplementary modular form to an orthocomplementary weakly modular lattice. The quantum logic was born. The next step was to find a useful mathematical presentation of this new logic. A historic review of what happened can be found in: "Quantum Theory: von Neumann" vs. Dirac; <http://www.ilic.uva.nl/~seop/entries/qt-nvd/>. It includes extensions of the concept of Hilbert space and application of these concepts to quantum field theory. Another source is: http://www.quantonics.com/Foulis_On_Quantum_Logic.html.

Quantum logic

Elementary particles behave non-classical. They can present themselves either as a particle or as a wave. A measurement of the particle properties of the object destroys the information that was obtained from an earlier measurement of the wave properties of that object. With elementary particles it becomes clear that that nature obeys a different logic than our old trusted classical logic. The difference resides in the modularity axiom. That axiom is weakened. The classical logic is congruent to an orthocomplemented modular lattice. The quantum logic is congruent to an orthocomplemented weakly modular lattice. Another name for that lattice is orthomodular lattice.

Lattices

A lattice is a set of elements a, b, c, \dots that is closed for the connections \cap and \cup . These connections obey:

- The set is partially ordered. With each pair of elements a, b belongs an element c , such that $a \subset c$ and $b \subset c$.
- The set is a \cap half lattice if with each pair of elements a, b an element c exists, such that $c = a \cap b$.
- The set is a \cup half lattice if with each pair of elements a, b an element c exists, such that $c = a \cup b$.
- The set is a lattice if it is both a \cap half lattice and a \cup half lattice.

The following relations hold in a lattice:

$$a \cap b = b \cap a \quad (\text{A1})$$

$$(a \cap b) \cap c = a \cap (b \cap c) \quad (\text{A2})$$

$$a \cap (a \cup b) = a \quad (\text{A3})$$

$$a \cup b = b \cup a \quad (\text{A4})$$

$$(a \cup b) \cup c = a \cup (b \cup c) \quad (\text{A5})$$

$$a \cup (a \cap b) = a \quad (\text{A6})$$

The lattice has a partial order inclusion \subset :

$$a \subset b \Leftrightarrow a \cap b = a \quad (\text{A7})$$

A complementary lattice contains two elements n and e with each element a a complementary element a' such that:

$$a \cap a' = n \quad (\text{A8})$$

$$a \cap n = n \quad (\text{A9})$$

$$a \cap e = a \quad (\text{A10})$$

$$a \cup a' = e \quad (\text{A11})$$

$$a \cup e = e \quad (\text{A12})$$

$$a \cup n = a \quad (\text{A13})$$

An orthocomplemented lattice contains two elements n and e and with each element a an element a'' such that:

$$a \cup a'' = e \quad (\text{A14})$$

$$a \cap a'' = n \quad (\text{A15})$$

$$(a'')'' = a \quad (\text{A15})$$

$$a \subset b \Leftrightarrow b'' \subset a'' \quad (\text{A16})$$

e is the unity element; n is the null element of the lattice

A distributive lattice supports the distributive laws:

$$a \cap (b \cup c) = (a \cap b) \cup (a \cap c) \quad (\text{A17})$$

$$a \cup (b \cap c) = (a \cup b) \cap (a \cup c) \quad (\text{A18})$$

A modular lattice supports:

$$(a \cap b) \cup (a \cap c) = a \cap (b \cup (a \cap c)) \quad (\text{A19})$$

A weak modular lattice supports instead:

There exists an element d such that

$$a \subset c \Leftrightarrow (a \cup b) \cap c = a \cup (b \cap c) \cup (d \cap c) \quad (\text{A20})$$

where d obeys:

$$(a \cup b) \cap d = d \quad (\text{A21})$$

$$a \cap d = n \quad (\text{A22})$$

$$b \cap d = n \quad (\text{A23})$$

$$[(a \subset g) \text{ and } (b \subset g)] \Leftrightarrow d \subset g \quad (\text{A24})$$

In an atomic lattice holds

$$\exists p \in L \forall x \in L \{x \subset p \Rightarrow x = n\} \quad (\text{A25})$$

$$\forall a \in L \forall x \in L \{(a < x < a \cap p) \Rightarrow (x = a \text{ or } x = a \cap p)\} \quad (\text{A26})$$

p is an atom

Both the set of propositions of quantum logic and the set of subspaces of a separable Hilbert space have the structure of an orthomodular lattice. In this respect these sets are congruent. In Hilbert space, an atom is a pure state (a ray spanned by a single vector).

Classical logic has the structure of an orthocomplemented distributive modular and atomic lattice.

Quantum logic has the structure of an orthomodular lattice. That is an orthocomplemented weakly modular and atomic lattice. The set of closed subspaces of a Hilbert space also has that structure.

Proposition

In Aristotelian logic a proposition is a particular kind of sentence, one which affirms or denies a predicate of a subject. Propositions have binary values. They are either true or they are false.

Propositions take forms like "*This is a particle or a wave*". In quantum logic "*This is a particle*." is not a proposition.

In mathematical logic, propositions, also called "propositional formulas" or "statement forms", are statements that do not contain quantifiers. They are composed of well-formed formulas consisting entirely of atomic formulas, the five [logical connectives](#), and symbols of grouping (parentheses etc.). Propositional logic is one of the few areas of mathematics that is totally solved, in the sense that it has been proven internally consistent, every theorem is true, and every true statement can be proved. Predicate logic is an extension of propositional logic, which adds variables and quantifiers.

In Hilbert space a vector is either inside or not inside a closed subspace. A proper quantum logical proposition is "*Vector $|f\rangle$ is inside state s* ".

In Hilbert space, an atomic proposition corresponds with a subspace that is spanned by a single vector.

Predicates may accept attributes and quantifiers. The predicate logic is also called first order logic. A dynamic logic can handle the fact that predicates may influence each other when atomic propositions are exchanged.

Observation

In physics, particularly in quantum physics, a system **observable** is a property of the system state that can be determined by some sequence of physical operations. This paper distinguishes between measurements and observations.

- With an observation the state is considered as a linear combination of eigenvectors of the observable. An observation returns the statistical expectation value of the eigenvalue of the observable.
- A measurement transforms the observed state to one of the eigenvectors of the observable. What happens depends on the characteristics of the measuring equipment. The measurement can be seen as a combination of a transformation and an observation.

Depending on the characteristics of the measuring equipment a measurement and a clean observation can give the same result.

With this interpretation of the concept of observation it is possible to let states observe other states. A state might do a transformation before doing an observation but in general it fails the equipment to arrange that transformation. In nature observations are far more common than measurements.

Dynamic logic

The current trend in quantum logic development is to add axioms that change the static character of quantum logic in a more dynamic and operational logic. Logic of quantum actions ([LQA](#)) adds unitary transforms as the source of dynamics. As we see in this article these transforms are not the real fundamental causes of dynamics. The fields that are exerted by the items are more fundamental causes of dynamics. They are the drivers behind the actions of the unitary transforms. Their existence can even be interpreted in logical terms. It is connected with the consequences of disorderly replacement of atomic propositions in an enveloping proposition. To my knowledge these influences are not yet covered by any dynamic logic theory.

More stuff

More useful stuff is collected in the [toolkit](#)

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